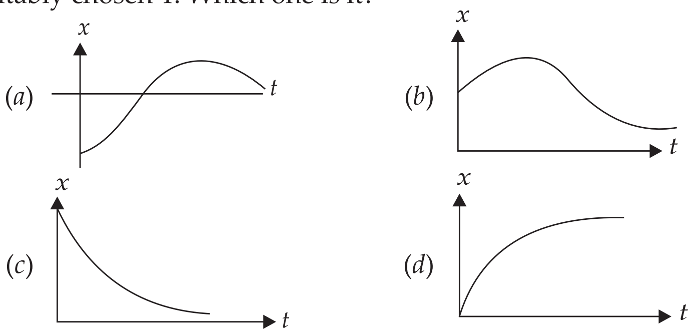
3 Motion in a Straight Line

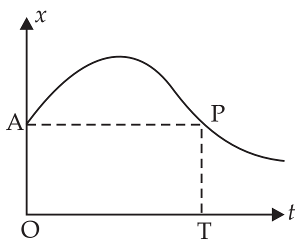
MULTIPLE CHOICE QUESTIONS-I

Q3.1. Among the four graphs given below, there is only one graph for which average velocity over the time interval (0, T) can vanish for a suitably chosen T. Which one is it?



Main concept used: Average velocity of body will be zero when displacement is zero any time interval-T in *x*-*t* graph.

Ans. (*b*): If we draw a line parallel to time axis from the point (A) on graph at $t = 0$ sec. This line can intersect graph again at P in only option (*b*) as shown in the figure.



The change in displacement (O-T) time is zero i.e., displacement at A and P are equal so as change in displacement zero so velocity of body vanishes to zero.

Q3.2. A lift is coming from 8th floor and is just about to reach 4th floor. Taking ground floor as origin and positive direction upwards for all quantities, which one of the following is correct?

(*a*) $x < 0, v < 0, a > 0$ (*b*) $x > 0, v < 0, a < 0$

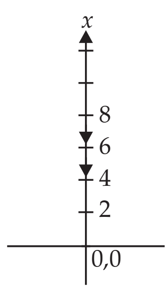
(*c*) $x > 0, v < 0, a > 0$ (*d*) $x > 0, v > 0, a < 0$

Ans. (*a*): As the lift moving downward so displacement is in negative or $\vec{x} < 0$.

As displacement is negative $\vec{v} < 0$. As the lift is just to reach 4th floor so it's motion is retarded ($-a$) downward (–).

So net acceleration $-(-a)$ = +ve i.e. $a > 0$.

Hence, verifies the option (*a*).



Q3.3. In one dimensional motion, instantaneous speed v satisfies $0 \le v < v_0$.

(*a*) The displacement in time T must always takes non-negative values.

(*b*) The displacement x in time T satisfies $-v_0 T < x < v_0 T$.

(*c*) The acceleration is always a non-negative number.

(*d*) The motion has no turning points.

Ans. (*b*): For maximum and minimum displacement we have the magnitude and direction of maximum velocity.

As maximum velocity in positive direction is v_0 maximum velocity in opposite direction is also v_0.

Maximum displacement in one direction = $v_0 T$

Maximum displacement in opposite direction = $-v_0 T$

Hence, $-v_0 T < x < v_0 T$.

Q3.4. A vehicle travels half the distance L with speed v_1 and the other half with speed v_2, then its average speed is

(*a*) $\dfrac{v_1 + v_2}{2}$ (*b*) $\dfrac{2v_1 + v_2}{v_1 + v_2}$ (*c*) $\dfrac{2v_1 v_2}{v_1 + v_2}$ (*d*) $\dfrac{L(v_1 + v_2)}{v_1 v_2}$

Ans. (*c*): Time t_1 taken in half distance $= t_1 = \dfrac{L}{v_1}$

Time t_2 taken in half distance $t_2 = \dfrac{L}{v_2}$

$$\text{Total time } (t) \text{ taken in distance } (L + L) = \frac{L}{v_1} + \frac{L}{v_2} = \frac{L(v_2 \quad v_1)}{v_1 v_2}$$

Total distance = L + L = 2L

$$\text{Average speed } v_{av} = \frac{\text{Total distance}}{\text{Total time}} = \frac{2L}{\dfrac{L(v_2 + v_1)}{v_1 v_2}} = \frac{2v_1 v_2}{(v_1 + v_2)}$$

Q3.5. The displacement of a particle is given by $x = (t-2)^2$ where x is in metres and t in seconds. The distance covered by the particle in first 4 seconds is:

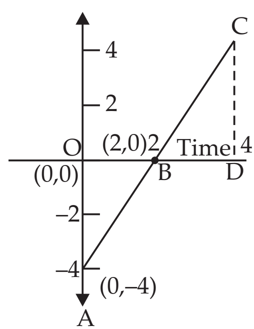
(*a*) 4 m (*b*) 8 m (*c*) 12 m (*d*) 16 m

Ans. (*b*):

$$x = (t-2)^2$$

$$v = \frac{dx}{dt} = 2(t-2) \text{ m/s}$$

$$a = \frac{d^2x}{dt^2} = 2(1-0) = 2 \text{ m s}^{-2}$$



at $t = 0$ $v_0 = 2(0-2) = -4$ m/s

$t = 2$ $v_2 = 2(2-2) = 0$ m/s

$t = 4$ $v_4 = 2(4-2) = 4$ m/s

Distance = Area between time axis and (v-t) graph

$$= \text{ar } \Delta OAB + \text{ar } \Delta BCD$$

$$= \frac{1}{2} OB \times OA + \frac{1}{2} BD \times CD$$

$$= \frac{1}{2} \times 2 \times 4 + \frac{1}{2} \times 2 \times 4$$

Distance = 8 m verifies option (*b*).

If displacement

$$= \frac{1}{2} OB \times OA + \frac{1}{2} BD \times CD$$

$$= \frac{1}{2} \times 2 \times (-4) + \frac{1}{2} \times 2 \times 4 = 0$$

Displacement = Zero.

Q3.6. At metro station, a girl walks up a stationary escalator in time t_1. If she remains stationary on escalator, then the escalator take her up in time t_2. The time taken by her to walk up on the moving escalator will be:

(*a*) $\frac{(t_1 + t_2)}{2}$ (*b*) $\frac{t_1 t_2}{(t_2 - t_1)}$ (*c*) $\frac{t_1 t_2}{(t_1 + t_2)}$ (*d*) $(t_1 - t_2)$

Ans. (*c*): Let L be the length of escalator.

Velocity of girl w.r.t. ground $v_g = \frac{L}{t_1}$

Velocity of escalator w.r.t. ground $v_e = \frac{L}{t_2}$

Velocity of girl on moving escalator with respect to ground

$$= v_g + v_e = \frac{L}{t_1} + \frac{L}{t_2} = L\left[\frac{1}{t_1} + \frac{1}{t_2}\right]$$

v_{gG} on moving escalator = V = $L\left[\frac{t_1 + t_2}{t_1 t_2}\right]$

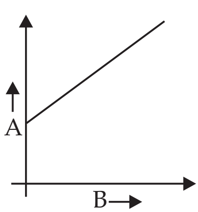
∴ Time t taken by girl on moving escalator in going up the distance L is

$$t = \frac{\text{distance}}{\text{speed}} = \frac{L}{L\left(\frac{t_1 + t_2}{t_1 t_2}\right)} = \frac{t_1 t_2}{t_1 + t_2}$$

Hence, verifies the option (*c*).

MULTIPLE CHOICE QUESTIONS-II

Q3.7. The variation of quantity A with quantity B, plotted in figure. Describe the motion of a particle in straight line.



(*a*) Quantity B may represent time.

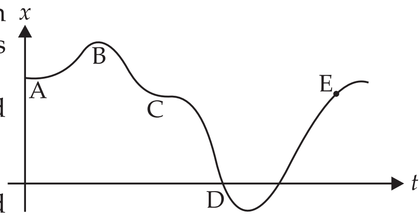
(*b*) Quantity A is velocity if motion is uniform.

(*c*) Quantity A is displacement if motion is uniform.

(*d*) Quantity A is velocity if motion is uniformly accelerated.

Ans. (*a, c, d*): If B represents time and A represents velocity then graph become (v-t). v-t graph is straight line so it is uniformly accelerated motion, so motion is not uniform. Verifies option (*a*), (*d*).
If B represents time and A represents displacement, then graph become (s-t) graph. Here s-t graph is straight line which represents uniform motion, so verifies the option (*c*).

Q3.8. A graph of x versus t shown in figure. Choose correct alternatives from below.



(*a*) The particle was released from rest at $t = 0$
(*b*) At B, the acceleration $a > 0$
(*c*) At C, the velocity and acceleration vanishes.
(*d*) Average velocity for the motion A and D is positive.
(*e*) The speed at D exceeds that at E.

Ans. (*a, c, e*): **Main concept used:** Slope of x-t graph gives $v = \frac{dx}{dt}$ At A graph (x-t) is parallel to time axis, so $\frac{dx}{dt}$ is zero or particle is at rest.
After A slope $\frac{dx}{dt}$ increases so velocity increases. Verifies option (*a*).
Tangent at B and C is graph (x-t) is parallel to time axis, so $\frac{dx}{dt} = 0$ or $v = 0$.
It implies that acceleration $a = 0$ so $a \ngtr 0$ discards option (*b*) and verifies the option (*c*).
From graph the slope at D is greater than at E. So speed at D is greater than at E. Verifies the option (*e*).
Velocity at A is zero as x-t parallel to time axis so average velocity at A is zero. At D displacement or slope is negative. So average velocity at D is negative not positive discards option (*d*).

Q3.9. For the one-dimensional motion, described by $x = t - \sin t$

(*a*) $x(t) > 0$ for all $t > 0$ (*b*) $v(t) > 0$ for all $t > 0$
(*c*) $a(t) > 0$ for all $t > 0$ (*d*) $v(t)$ lies between 0 and 2

Ans. (*a, d*):

$$\boxed{x = t - \sin t}$$

$$v = \frac{dx}{dt} = (1 - \cos t) \quad \Rightarrow \quad \boxed{v = (1 - \cos t)}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(1 - \cos t) = + \sin t$$

$$\boxed{a = \sin t}$$

For v_{max} at $\cos t$ minimum i.e., $\cos t = -1$.

$\therefore \quad v_{max} = 1 - (-1) = 2$

For v_{min} at cos t maximum i.e., cos $t = 1$

$$v_{min} = 1 - 1 = 0$$

Hence, v lies between 0 to 2. Verifies the option (*d*).

$$x = t - \sin t$$

sin t varies between 1 and – 1 for $t > 0$.

x will be always positive $x(t) > 0$. Verifies answer (*a*).

$$v = 1 - \cos t$$

cos t also varies from – 1 to 1,

at $\cos t = +1$

$$v = 1 - 1 = 0$$

$v(t) > 0$ is false discards option (*b*).

$$a = \sin t$$

sin t varies from – 1 to 1.

So a will varies from – 1 to 1 or a can be (–). So discards option (*c*).

Hence, verifies option (*a*) and (*d*).

Q3.10. A spring with one end attached to a mass m and other end to a rigid support is stretched and released.

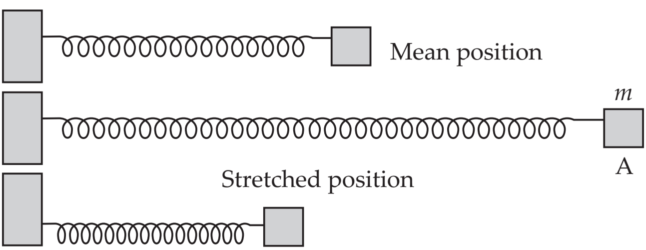
(*a*) Magnitude of acceleration, when just released is maximum.

(*b*) Magnitude of acceleration, when at equilibrium position is maximum.

(*c*) Speed is maximum when mass is at equilibrium position.

(*d*) Magnitude of displacement is always maximum, whenever speed is minimum.

Ans. (*a, c, d*): Consider a spring of spring constant k is attached to mass m at one end and other end is fixed at right support. Spring is lying on a frictionless table.



Now spring is stretched by a force F by x displacement then $F = -kx$ (–) sign shows that displacement x is opposite to the direction of force applied, when a force F acts on spring also applies equal to opposite force. P.E. at A = $\frac{1}{2}kx^2$ the restoring force is directly proportional to the x so execute Simple Harmonic Motion (SHM)

$$\therefore \quad a = \frac{-F}{m}$$

$$a = \frac{-kx}{m}$$

at $x = 0$ then $a = 0$

at $x = x$, $a = \frac{-kx}{m}$

Magnitude of a is maximum at x when released. Verifies the option (*a*).

At mean position where P.E. is converted into KE = $\frac{1}{2}mv^2$.

So the speed of mass is maximum at $x = 0$.

Verifies the option (*c*).

Magnitude of $a = 0$ at $x = 0$. So (*b*) is incorrect when mass (m) is at its maximum displacement then it returns at this point and momentarily $v = 0$. So it verifies answer (*d*) also.

Q3.11. A ball is bouncing elastically with a speed of 1 m/s between walls of a railway compartment of size 10 m in the direction perpendicular to walls. The train is moving at a constant speed of 10 m/s parallel to the direction of motion of the ball. As seen from the ground,

(*a*) the direction of motion of the ball changes every 10 seconds.
(*b*) speed of the ball changes every 10 seconds.
(*c*) average speed of the ball over any 20 seconds interval is fixed
(*d*) the acceleration of ball is the same as from the train.

Main concept used: Motion of ball with respect to observer and relative velocity of body.

Ans. (*a, c, d*): As the motion is observed from ground, time to strike ball with walls will be after every 10 sec. So the direction speed of ball changes (direction) every 10 sec, but change in speed is zero.

i.e., speed of ball remains always 1 m/s so option (*a*) is correct and (*b*) is incorrect.

As speed of ball is uniform so average speed at any time remain same or 1 m/s with respect to train or ground. So option (*c*) is correct.

Speed of ball changes when it strike to wall initial speed of ball in the direction of moving train with respect to ground = 10 + 1 = 11 m/s.

v_{BG} (opposite to the direction of train) = 10 – 1 = 9 m/s.

$\therefore$ Change in velocity on collision will be in magnitude = 11 – 9 = 2 m/s.

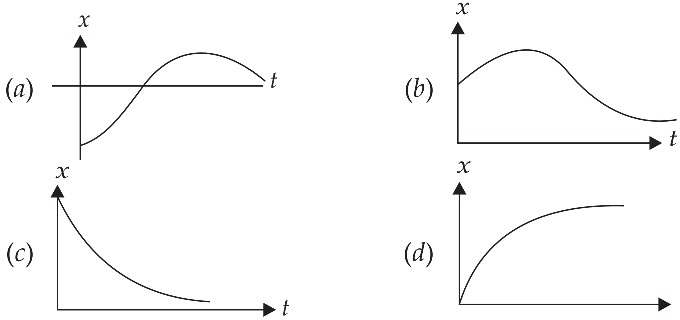
So magnitude of acceleration on both walls of compartment is same but direction will be opposite. Hence, right option are (*a, c, d*).

VERY SHORT ANSWER TYPE QUESTIONS

Q3.12. Refer to graph of Question 3.1. Match the following.

Graph	Characteristic
(*a*)	(*i*) has $v > 0$ and $a < 0$ throughout.
(*b*)	(*ii*) has $x > 0$ throughout and has a point with $v = 0$, and a point $a = 0$.
(*c*)	(*iii*) has a point with zero displacement for $t > 0$.
(*d*)	(*iv*) has $v < 0$ and $a < 0$

Ans.



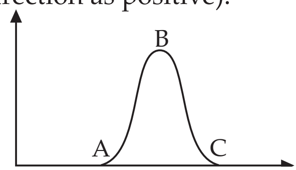
(*i*) $v > 0$ indicate that slope is always positive i.e., between 0° to 90° (tan θ) it matches with graph (*d*). Hence, (*i*) part matches with graph (*d*).

(*ii*) $x > 0$ throughout and $v = 0$, $v = 0$ matches with graph (*ii*). At point A slope is zero so $v = 0$, $a = 0$ graph lies in + x direction always so verifies the answer. So (*ii*) part matches with (*b*).

(*iii*) Zero displacement where $y = 0$ is only in graph (*a*). So part (*iii*) matches with graph (*a*).

(*iv*) $v < 0$ i.e. slope is (–)ve it is in graph (*c*). So part (*iii*) matches with graph (*c*).

Q3.13. A uniformly moving cricket ball is turned back by hitting it with a bat for a very short time interval. Show the variation of its acceleration with time. (Take acceleration in backward direction as positive).



Ans. When ball is hitted by bat its acceleration decreases till its velocity becomes zero, so acceleration is in **backward** direction which here taken is positive as shown in graph from A to B part.

Now after when the velocity of ball decreased to zero its velocity increases in forward direction so acceleration in forward direction is negative (here) show by part BC in graph.

Q3.14. Give examples of a one dimensional motion where

(*a*) the particle moving along positive *x*-direction comes to rest periodically and moves *forward*.

(*b*) the particle moving along positive *x*-direction comes to rest periodically and moves *backward*.

Ans. (*i*) Consider a motion $x(t) = \omega t - \sin \omega t$

$$v = \frac{dx}{dt} = \omega - \omega \cos \omega t$$

$$a = \frac{dv}{dt} = \omega^2 \sin \omega t$$

at $\omega t = 0$ $\quad x(t) = 0;$ $\quad v = 0$ $\quad a = 0$

at $\omega t = \pi$ $\quad x(t) = \pi > 0;$ $\quad v = \omega - \omega\cos\pi = 2\omega > 0$ $\quad a = 0$

at $\omega t = 2\pi$ $\quad x(t) = 2\pi > 0;$ $\quad v = 0$ $\quad a = 0$

(*ii*) Consider a function of motion

$$x(t) = -a\sin\omega t$$

at $t = 0$ $\quad x(t) = -a\sin 0 = 0$

at $t = \frac{T}{4}$ $\quad x(t) = -a\sin\frac{2\pi}{T}.\frac{T}{4} = -a\sin\frac{\pi}{2} = -a$

at $t = \frac{T}{2}$ $\quad x(t) = -a\sin\frac{2\pi}{T}.\frac{T}{2} = -a\sin\pi = 0$

at $t = \frac{3T}{4}$ $\quad x(t) = -a\sin\frac{2\pi}{T}.\frac{3T}{4} = -a\sin\frac{3\pi}{2} = -a\sin\left(\pi + \frac{\pi}{2}\right)$

$$= -a\left(-\sin\frac{\pi}{2}\right) = a$$

at $t = T$ $\quad x(t) = -a\sin\frac{2\pi}{T}.T = -a\sin 2\pi = +0$

Hence, the particle comes to rest periodically and displacement is in negative direction.

Hence, periodic function is $-a\sin\omega t$

$$v = \frac{dx(t)}{dt} = \frac{d}{dt}(-a\sin\omega t) = -a\omega\cos\omega t$$

at, $t = 0$ $\quad v = -a\omega\cos 0° = -\omega a$

at, $t = \frac{T}{4}$ $\quad v = -\omega a\cos\frac{2\pi}{T}.\frac{T}{4} = -\omega a\cos\frac{\pi}{2} = 0$

at, $t = \frac{T}{2}$ $\quad v = -\omega a\cos\frac{2\pi}{T}.\frac{T}{2} = -a\omega\cos\pi = +\omega a$

at, $t = \frac{3T}{4}$ $\quad v = -\omega a\cos\frac{2\pi}{T}.\frac{3T}{4} = -\omega a\cos\frac{3\pi}{2} = -\omega a\cos\left(\pi + \frac{\pi}{2}\right)$

$$v = +a\omega\cos\frac{\pi}{2} = a\omega \times 0 = 0$$

at, $t = T$ $\quad v = -a\omega\cos\frac{2\pi}{T}.T = -\omega a\cos 2\pi = -\omega a$

Hence, the velocity after zero displacement changes periodically.

So required function of motion $= x(t) = -a\sin\omega t$.

(*i*) Consider a function of motion of time periods and amplitude a, $x(t) = a\sin\omega t$.

$$x(0) = 0 = 0$$

$$x\left(\frac{T}{4}\right) = a\sin\frac{2\pi}{T}.\frac{T}{4} = a\sin\frac{\pi}{2} = a$$

$$x\left(\frac{T}{2}\right) = a\sin\frac{2\pi}{T}.\frac{T}{2} = a\sin\pi = 0$$

$$x\left(\frac{3T}{2}\right) = a\sin\frac{2\pi}{T}.\frac{3T}{4} = a\sin\frac{3\pi}{2} = a\sin\left(\pi + \frac{\pi}{2}\right) = -a$$

$$x(\text{T}) = a \sin \frac{2\pi}{\text{T}}.\text{T} = a \sin 2\pi = 0$$

Hence, particle is moving with displacement zero periodically and moves in +ive direction i.e., in forward.

Hence, require function is $x(t) = a \sin \omega t$.

Q3.15. Give example of a motion where $x > 0$, $v < 0$ and $a > 0$ at a particular instant.

Ans. Let us consider function of motion

$$x(t) = \text{A} + \text{B}e^{-\gamma t} \quad \text{...}(i)$$

where γ and A, is a constant B is amplitude

$x(t)$ is displacement at time t, where $\text{A} > \text{B}$ and $\gamma > 0$

$$v(t) = \frac{dx(t)}{dt} = 0 + (-\gamma)\,\text{B}e^{-\gamma t} = -\gamma\,\text{B}e^{-\gamma t} \quad \text{...}(ii)$$

$$a(t) = \frac{d[v(t)]}{dt} = \frac{d}{dt}(-\gamma\,\text{B}e^{-\gamma t}) = +\gamma^2\text{B}e^{-\gamma t} \quad \text{...}(iii)$$

From (i) $\because$ A > B so x is always +ve i.e., $x > 0$

From (ii) v is always negative from (ii) $v < 0$

From (iii) a is always again positive $a > 0$

As the value of $\gamma^2\text{B}e^{-\gamma t}$ can varies from 0 to $+\infty$.

Q3.16. An object falling through a fluid is observed to have acceleration given by $a = g - bv$ where g = gravitational acceleration and b is a constant. After a long time of release, it is observed to fall with constant speed. What must be the value of constant speed?

Ans. After long time of released the velocity becomes constant i.e.,

$$\frac{dv(t)}{dt} = 0 \quad \text{or} \quad a = 0 \quad \text{...}(i)$$

Given acceleration is

$$a = g - bv$$

$$0 = g - bv \quad [\text{from } (i)]$$

$$bv = g$$

$$v = \frac{g}{b}$$

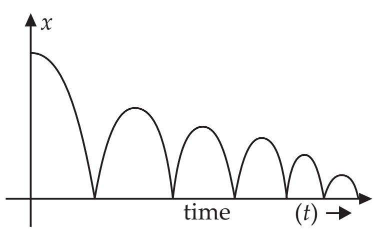
Hence, the constant speed after long time of release is $\left(\frac{g}{b}\right)$.

SHORT ANSWER TYPE QUESTIONS

Q3.17. A ball is dropped and its displacement verses time graph is shown (Displacement x is from ground and all quantities are positive upwards.)

(a) Plot qualitatively velocity verses time graph.

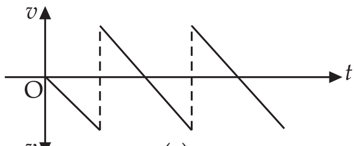
(b) Plot qualitatively acceleration verses time graph.



Ans. It is clear from graph, displacement (x) is always positive. Velocity of body increases till the x becomes zero then velocity becomes in

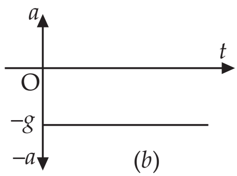
opposite direction and velocity (slope of x-t graph) decreases to zero till it reaches maximum value of x but smaller than earlier.
When velocity increases and body reaches towards $x = 0$ acceleration is in downward direction. When body moves upward i.e., $x > 0$ then velocity decreases so direction of 'a' is again downward.
Hence the $a = -g$ always.

(*a*) at $t = 0$, $v = 0$ it increases in downward direction with constant accele-ration 'g'.
When $x = 0$ body after it bounces upward but its velocity decreases with constant $a = g$ if it again come back (downward) with acceleration ($-g$). So v-t graph.



(*a*)

(*b*) The (a-t) graph



(*b*)

Q3.18. A particle length executes the motion described by $x(t) = x_0(1 - e^{-\gamma t})$; $t \geq 0$, $x_0 > 0$

(*a*) where does the particle start and with what velocity?
(*b*) find the maximum and minimum values of $x(t)$, $v(t)$, $a(t)$. Show that $x(t)$ and $a(t)$ increases with time and $v(t)$ decreases with time.

Main concept used: By calculating $v(t)$ and $a(t)$ with the help of $x(t)$, then determining the maximum and minimum value of $x(t)$, $v(t)$ and $a(t)$.

Ans.
$$x(t) = x_0\left[1 - e^{-\gamma t}\right] \quad ...(i)$$
$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[x_0(1 - e^{-\gamma t})\right] = +x_0\gamma\, e^{-\gamma t} \quad ...(ii)$$
$$a(t) = \frac{dv}{dt} = \frac{d\left[+x_0\,\gamma\, e^{-\gamma t}\right]}{dt} = -x_0\,\gamma^2\, e^{-\gamma t} \quad ...(iii)$$

(*i*) At, $t = 0$
$$x(0) = x_0\left[1 - e^0\right] = x_0(1 - 1) = 0$$
$$v(0) = x_0\,\gamma\, e^0 = x_0\,\gamma$$
Hence, the particle start from $x = 0$ with velocity $v_0 = x_0\,\gamma$.

(*ii*) (*a*) $x(t)$ is minimum at $t = 0$
$x(t)$ is maximum $t = \infty$ $\quad \because e^{-\gamma t} = \infty$ at $t = \infty$.

(*b*) $v(t)$ at $t = 0$ is v_0 $(= x_0\,\gamma)$ (maximum)
at $t = \infty$ $\quad v(t) = x_0\,\gamma\,(0) = 0$ (minimum)
$a(t)$ at $t = 0$ is $a(0) = -x_0\,\gamma^2$ (minimum)
$a(t)$ at $t = \alpha$ is $a(\infty) = 0$ (maximum)

Q3.19. A bird is tossing (flying to and fro) between two cars moving towards each other on a straight road. One car has a speed of 18 km/hr while the other has the speed of 27 km/hr. The bird starts moving from first car towards the other and is moving with the speed of 36 km/hr and when the two cars were separated by 36 km. What is the total distance covered by the bird? What is the total displacement of the bird?

Main concept used: Bird will fly to and fro till both the cars meet together. So the total distance covered by bird during the time speed of bird × time to meet the cars together.

Ans. Time to meet the two cars together (t)

$$t = \frac{\text{distance between cars}}{\text{relative speed of cars}} = \frac{36 \text{ km}}{(27+18) \text{ km/hr}} = \frac{36}{45} = \frac{4}{5} \Rightarrow t = \frac{4}{5} \text{ hours}$$

$\therefore$ Distance covered by bird in $\frac{4}{5}$ hours $= 36 \times \frac{4}{5} = 28.8$ km.

Q3.20. A man runs across the roof-top of a tall building and jumps horizontally with the hope of landing on the roof of the next building which is of a lower height than the first. If his speed is 9 m/s, the (horizontal) distance between the two buildings is 10 m and the height difference is 9 m, will he be able to land on the next building? (take $g = 10$ m/s^2)

Main concept: During fall freely 9 m the horizontal distance covered by man should be atleast 10 m.

Ans. Vertical motion

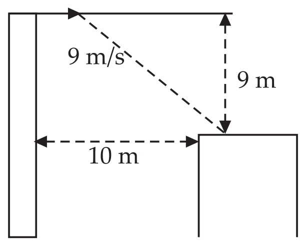
$u_y = 0,\quad a = 10$ m/s^2

$s = 9$ m $\quad t = t$

$$s = u_y t + \frac{1}{2}at^2$$

$$9 = 0 \times t + \frac{1}{2} \times 10 \times t^2$$

$$t = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}} \text{ sec}$$



Horizontal distance covered by person is

$$\frac{3}{\sqrt{5}} \text{ sec} = 9 \text{ m/s} \times \frac{3}{\sqrt{5}} \text{ sec} = \frac{27}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{27 \times 2.236}{5} = \frac{60.37}{5} = 12.07 \text{ m}$$

As 12.07 m covered during free falling of 9 m. So he reach on the building next farther the first edge by 12.07 – 10 = 2.07 m.

Q3.21. A ball is dropped from a building of height 45 m. Simultaneously another ball is thrown up with a speed 40 m/s. Calculate the relative speed of the balls as a function of time.

Ans. For the first ball falling from top

$v = v_1 = ?\quad u = 0\quad h = 45$ m $\quad a = g\quad t = t$

$$v = u + at$$

$v_1 = 0 + gt$ or $v_1 = gt$ downward $\therefore v_1 = -gt$

For the second ball thrown upward

$v = v_2\quad u = 40$ m/s $\quad a = -g\quad t = t$

$\therefore\quad v = u + at$

$$v_2 = (40 - gt) \quad \text{upward} \quad \therefore\ v_2 = +(40 - gt)$$

Relative velocity of ball Ist with respect to IInd

$$v_{12} = v_1 - v_2 = -gt - (40 - gt)$$
$$= -gt - 40 + gt = -40 \text{ m/s (downward)}$$

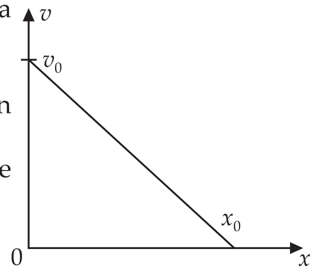
Relative velocity of ball first with the respect to second is 40 m/s downward.

In this problem due to acceleration the speed of one increases and of other decreases with same rate. So their relative speed remains (40 – 0) = 40 m/s.

Q3.22. The velocity-displacement graph of a particle is shown in figure.

(*a*) Write the relation between v and x.

(*b*) Obtain the relation between acceleration and displacement and plot it.

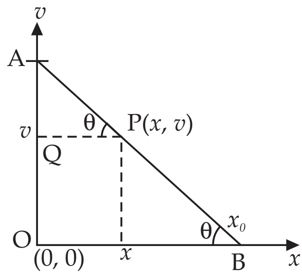


Ans. (*a*) Consider a point P(x, v) at any time t on graph. Let ∠ABO is θ then

$$\tan\theta = \frac{AQ}{QP} = \frac{v_0 - v}{x} = \frac{v_0}{x_0}$$

As velocity decrease from v_0 to zero during displacement zero to x.

So acceleration is negative



$$a = -\tan\theta = \frac{-(v_0 - v)}{x} = \frac{-v_0}{x_0}$$

$$v_0 - v = \frac{v_0}{x_0} x$$

$$v = v_0 - \frac{v_0}{x_0}.x$$

$$v = v_0\left[1 - \frac{x}{x_0}\right] \text{ is the relation between } v \text{ and } x.$$

(*b*)
$$a = \frac{dv}{dt} = \frac{dv}{dt} \times \frac{dx}{dx} = \frac{dv}{dx}.\frac{dx}{dt}$$

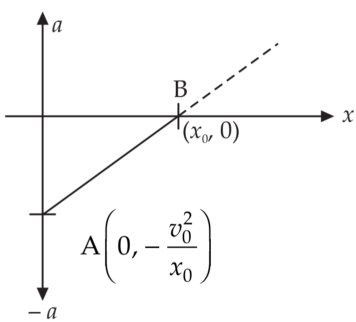
$$a = \frac{-v_0}{x_0}.v = \frac{-v_0}{x_0}.\left[v_0\left(1 - \frac{x}{x_0}\right)\right]$$

$$= \frac{-v_0^2}{x_0}\left(1 - \frac{x}{x_0}\right)$$

$$a = \frac{v_0^2 x}{x_0^2} - \frac{v_0^2}{x_0}$$

at $x = 0$
$$a = \frac{v_0^2}{x_0^2} \times 0 - \frac{v_0^2}{x_0}$$

$$a = -\frac{v_0^2}{x_0}$$



at $a = 0$ $\quad 0 = \frac{v_0^2}{x_0^2}x - \frac{v_0^2}{x_0}$

$$\frac{v_0^2}{x_0} = \frac{v_0^2}{x_0^2}x \Rightarrow x = x_0$$

$\therefore$ A$\left(0, \frac{-v_0^2}{x_0}\right)$ and B$(x_0, 0)$.

LONG ANSWER TYPE QUESTIONS

Q3.23. It is a common observation that rain clouds can be at about a kilometre altitude above the ground.

(*a*) If a rain drop falls from such a height freely under gravity, what will be its speed? Also calculate in km/h. ($g = 10$ m/s^2)

(*b*) A typical rain drop is about 4 mm diameter. Momentum is mass × speed in magnitude. Estimate its momentum when it hits ground.

(*c*) Estimate the time required to flatten the drop.

(*d*) Rate of change of momentum is force. Estimate how much force such a drop would exert on you.

(*e*) Estimate the order of magnitude force on umbrella. Typical lateral separation between two rain drops is 5 cm.
(Assume that umbrella is circular and has a diameter of 1 m and cloth is not pierced through!!)

Main concept used: F = ma, $\frac{dp}{dt}$ = F, eqn. of motion, $p = mv$

Ans. Given $\quad h = 1$ km = 1000 m, $\quad g = 10$ m/s^2

$u = 0$ m/s $\quad d = 4$ mm $\Rightarrow r = \frac{4}{2}$ mm $= 2 \times 10^{-3}$ m

(*a*) Velocity of rain drop on ground

$$v^2 = u^2 + 2gh$$
$$v^2 = 0^2 + 2 \times 10 \times 1000$$
$$v = 100\sqrt{2} \text{ m/s} \quad \text{Ans. } (i)$$
$$v = \overset{20}{\cancel{100}}\sqrt{2} \times \frac{18}{\cancel{5}} \text{ km/hr} = 360\sqrt{2} \text{ km/hr} \quad \text{Ans. } (ii)$$

(*b*) Mass of drop = Volume × density = $\frac{4}{3}\pi r^3 \rho$

$$= \frac{4}{3}\pi(2 \times 10^{-3})^3 \times 1000 \text{ kg/m}^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2 \times 10^{-9} \times 10^3$$
$$= \frac{32 \times 22}{21} \times 10^{-6} \text{ kg}$$
$$m = \frac{704}{21} \times 10^{-6} = 33.5 \times 10^{-6} \text{ kg}$$

$\therefore$ Momentum $= mv = 33.5 \times 10^{-6} \times 100\sqrt{2}$

$= 33.5 \times 1.414 \times 10^{-4}$ kg ms^{-1}

$= 47.37 \times 10^{-4}$ kg ms^{-1}

$= 4.7 \times 10^{-3}$ kg ms^{-1}

(*c*) Time to required a drop of 4 mm diameter spherical drop i.e., time to reach upper part of spherical drop on ground i.e., distance covered by upper part of drop to reach ground = diameter (d) = 4 mm = 4×10^{-3} m

$$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{4 \times 10^{-3}\ \text{m}}{100\sqrt{2}} = \frac{\overset{2}{\cancel{4}} \times \sqrt{2}}{100 \times \cancel{2}} \times 10^{-3}$$

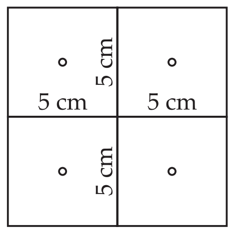
$$\text{Time } (t) = \frac{2 \times 1.414}{100} \times 10^{-3} = \frac{2.828}{100} \times 10^{-3} = 2.8 \times 10^{-5} \text{ sec}$$

(*d*)
$$\text{Force} = \frac{dp}{dt} = \frac{mv - 0}{t - 0} = \frac{4.7 \times 10^{-3}}{2.8 \times 10^{-5}} = 1.68 \times 10^{2}$$

$= 168$ N **Ans.**

(it equivalent to 16 kg which is not actually force exerted by drop on ground or man)

(*e*) Area of umbrella $= \pi R^2 = \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2}$ m^2



Square area covered by one drop

$= (5 \times 10^{-2})^2 = 25 \times 10^{-4}$ m^2

Number of drops falling on umbrella

$$= \frac{\pi R^2}{25 \times 10^{-4}}$$

$$= \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{25 \times 10^{-4}} \text{ drops}$$

$$= \frac{11 \times 10^4}{14 \times 25} = \frac{11 \times 10^4}{350} = 0.0314 \times 10^4$$

$n = 314$ drops

$\therefore$ Net force on umbrella by 314 drops $= 314 \times 168$ N $= 52752$ N

It is equivalent to 5,275 kg wt which again not possible on umbrella. Velocity of drop decreased to terminal velocity due to retarding force of friction of air molecules.

Q3.24. A motor car moving at a speed of 72 km/h cannot come to stop in less than 3.0 second while for a truck this time interval is 5.0 second. On a highway the car is behind the truck both moving 72 km/h. The truck gives a signal that it is going to stop at emergency. At what distance the car should be from the truck so that it does not bump onto (collide with) the truck? Human response time is 0.5 s.

(Comment: This is to illustrate why vehicles carry the message on the rear side. "Keep Safe Distance").

Ans. For truck $u = 72 \times \frac{5}{18}$ m/s = 20 m/s

$v = 0 \qquad a = ? \qquad t = 5$ sec.

$$v = u + at$$
$$0 = 20 + a_T \times 5$$
$$a_T = \frac{-20}{5} = -4 \text{ m/s}^2$$

For car $\quad t = 3$ s $\quad u = 20$ m/s $\quad v = 0 \quad a = a_C$

$$v = u + at$$
$$0 = 20 + a_C 3$$
$$a_C = \frac{-20}{3} \text{ m/s}^2$$

Let car is at distance x metre behind the truck. Car takes time 't' to stop after observing the signal given by truck to stop.

Time of response for human = 0.5 second

Time t includes the time to stop car and responding time both. So time taken by car to stop after applying breaks is (t – 0.5) seconds.

$$v_C = u + a_c t$$
$$0 = 20 - \frac{20}{3}(t - 0.5) \qquad ...(i)$$

For truck driver there is no responding time he applies breaks with passing signal to car back side, so

$$v_T = u + at$$
$$0 = 20 - 4t \qquad ...(ii)$$

Equating (i) and (ii) equation.

$$20 - 4t = 20 - \frac{20}{3}(t - 0.5)$$
$$-4t = -\frac{20}{3}(t - 0.5)$$
$$12t = 20t - 10$$
$$-20t + 12t = -10$$
$$-8t = -10$$
$$t = \frac{10}{8} = \frac{5}{4} = 1.25 \text{ seconds}$$

Distance travelled by car and truck in $\frac{5}{4}$ sec

$$s = 20 \times \frac{5}{4} + \frac{1}{2}(-4)\frac{5}{4} \times \frac{5}{4} \qquad \left(\because s = ut + \frac{1}{2}at^2\right)$$
$$s_T = 25 - \frac{25}{8} = 25 - 3.125 = 21.875 \text{ m.}$$

Car travel first 0.5 sec with speed of uniform but after this responding time 0.5 sec breaks are applied and then retarding motion starts for car

$$s_C = (20\times .5) + 20(1.25-0.5) + \frac{1}{2}\times\left(\frac{-20}{3}\right)(1.25-0.5)^2$$
$$= 10 + 20\times 0.75 - \frac{10}{3}\times 0.75\times 0.75 = 10 + 15.0 - 7.5\times .25$$
$$s_C = 25 - 1.875 = 23.125 \text{ m}$$
$$s_C - s_T = 23.125 - 21.875 = 1.25 \text{ m.}$$

So avoid bump onto truck, the car must be behind atleast 1.25 m.

Q3.25. A monkey climbs up a slippery pole for 3 seconds and subsequently slips for 3 seconds. Its velocity at time 't' is given by

$$v(t) = 2t\,(3-t); \quad 0 < t < 3 \text{ seconds}$$

and $v(t) = -(t-3)\,(6-t)$ for $3 < t < 6$ s in m/s. It repeats this cycle till it reaches the height of 20 m.

(*a*) At what time its velocity is maximum?

(*b*) At what time its average velocity is maximum?

(*c*) At what time its acceleration is maximum in magnitude?

(*d*) How many cycles (counting fractions) are required to reach the top?

Ans. (*a*) For maximum velocity $v(t)$

$$\frac{dv(t)}{dt} = 0$$
$$\frac{d[2t(3-t)]}{dt} = 0$$
$$\frac{d(6t-2t^2)}{dt} = 0$$
$$6 - 4t = 0$$
$$4t = 6 \quad \Rightarrow \quad t = \frac{6}{4} = \frac{3}{2} = 1.5 \text{ second} \quad \textbf{Ans.}$$

(*b*) For average velocity $= \dfrac{\text{Total distance}}{\text{Total time}}$

$$\because \quad v(t) = 6t - 2t^2 \qquad \ldots(i)$$
$$\frac{ds(t)}{dt} = 6t - 2t^2$$
$$ds = (6t - 2t^2)\,dt$$

Integrating B.S. from 0 to 3 sec

$$\int_0^s ds = \int_0^3 (6t - 2t^2)\,dt$$
$$s = \left[6\frac{t^2}{2} - 2\times\frac{t^3}{3}\right]_0^3 = \left[3t^2 - \frac{2}{3}t^3\right]_0^3$$
$$= \left[3\times 9 - \frac{2}{3}\times 27\right] = 27 - 18$$
$$s = 9 \text{ m} \qquad \ldots(ii)$$

Average velocity $v_{av} = \frac{9 \text{ m}}{3} = 3$ m/s

$v(t) = 6t - 2t^2$ (given $0 < t < 3$)

$3 = 6t - 2t^2$ $v_{av} = 3$

$2t^2 - 6t + 3 = 0$ $a = 2$, $b = -6$, $c = 3$

$$t = \frac{+6 \pm \sqrt{(-6)^2 - 4(2)(3)}}{2 \times 2}$$

$$t = \frac{6 \pm \sqrt{36 - 24}}{2 \times 2} = \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4}$$

$$t = \frac{2(3 \pm \sqrt{3})}{4} = \frac{3 \pm \sqrt{3}}{2}$$

[Taking +ve] $t = \frac{3 + 1.732}{2} = \frac{4.732}{2} = 2.31$ sec

[Taking –ve] $t = \frac{3 - 1.732}{2} = \frac{1.268}{2} = .634$ sec rejected.

As $v = 3$ is not equal to 3 m/s from (*i*).

So average velocity is maximum at 2.31 sec.

(*c*) Time for maximum acceleration in periodic motion acceleration is maximum when body returns at its mean position or changes the direction of motion it at $v = 0$

$v(t) = 6t - 2t^2$

For maximum acceleration $v = 0$

$0 = 6t - 2t^2$

$2t(3 - t) = 0$

$t \neq 0$ $\therefore$ at $t = 3$ second acceleration is maximum.

(*d*) Distance covered from 0-3 sec

$s = 9$ m [from (*ii*) in (*b*) part]

For 3 to 6 seconds

$v(t) = -(t - 3)(6 - t)$

$\frac{ds}{dt} = (t - 3)(t - 6)$

$ds = (t^2 - 9t + 18)\, dt$

Integrating both sides from 3 s – 6 s

$$s_2 = \int_3^6 (t^2 - 9t + 18)\, dt = \left[\frac{t^3}{3} - \frac{9}{2}t^2 + 18t\right]_3^6$$

$$s_2 = \frac{(6)^3}{3} - \frac{9}{2}(6)^2 + 18 \times 6 - \left[\frac{(3)^3}{3} - \frac{9}{2}(3)^2 + 18 \times 3\right]$$

$$= \frac{6 \times 6 \times 6}{3} - \frac{9 \times 6 \times 6}{2} + 108 - \frac{3 \times 3 \times 3}{3} + \frac{9 \times 3 \times 3}{2} - 54$$

$$= 72 - 162 + 108 - 9 + \frac{81}{2} - 54$$

$$= 180 - 162 - 63 + 40.5 = 18 - 22.5$$
$$s_2 = -4.5 \text{ m}$$

s_2 distance is downward $\therefore s_2 = -4.5$ m
So net distance = 9 – 4.5 = 4.5 m.
Height climb up in three cycles = $4.5 \times 3 = 13.5$ m
Now remaining height = 20 – 13.5 = 6.5 m
Remaining height to climb is 6.5 m but monkey can climb 9 m up without slip. So in 4th cycle it will slip as it reaches on the top of pole.
Net number of cycle to climb 20 m high pole is 4.

Q3.26. A man is standing on the top of building 100 m high. He throws two balls vertically, one at $t = 0$ and other after a time interval (less than 2 seconds). The later ball is thrown at a velocity of half the first. The vertical gap between first and second ball is + 15 m at $t = 2$ s. The gap is found to remain constant. Calculate the velocity with which balls were thrown and the exact time interval between their throw.

Ans. Let the speed of ball 1 = $u_1 = 2u$ m/s
Then the speed of ball 2 = $u_2 = u$ m/s
Let the height covered by ball 1 before coming to rest = h_1
Let the height covered by ball 2 before coming to rest = h_2

$$\because \quad v^2 = u^2 + 2gh$$

At top their velocities becomes zero

$$\therefore \quad v^2 = 2gh \quad \text{or} \quad h = \frac{u^2}{2g}$$

$$\Rightarrow \quad h_1 = \frac{u_1^2}{2g} = \frac{4u^2}{2g} \quad \text{and} \quad h_2 = \frac{u^2}{2g}$$

According to question $h_1 - h_2 = 15$ m (given)

$$\therefore \quad \frac{4u^2}{2g} - \frac{u^2}{2g} = 15 \quad \text{(given)}$$

$$\frac{u^2}{2g}[4-1] = 15$$

$$u^2 = \frac{15 \times 2 \times 10}{3} \quad \Rightarrow \quad u = 10 \text{ m/s}$$

$$h_1 = \frac{4 \times 10 \times 10}{2 \times 10} = 20 \text{ m} \qquad h_2 = \frac{10 \times 10}{2 \times 10} = 5 \text{ m}$$

For 1st ball $\quad v_1 = u_1 + gt$

$$0 = 20 - 10t_1 \quad \Rightarrow \quad t_1 = 2 \text{ sec}$$

For ball 2 $\quad v_2 = u_2 + gt_2$

$$0 = 10 - 10t_2 \quad \Rightarrow \quad t_2 = 1 \text{ sec}$$

Velocities of ball 1 and 2 are 20 m/s and 10 m/s.
Exact time intervals between 2 balls = $t_1 - t_2 = (2 - 1) = 1$ second.

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