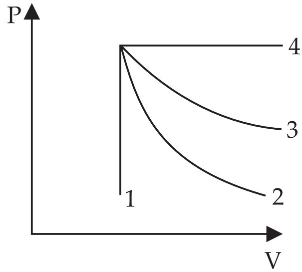
12 Thermodynamics

MULTIPLE CHOICE QUESTIONS-I

Q12.1. An ideal gas undergoes four different processes from the same initial state (figure). Four processes are adiabatic, isothermal, isobaric and isochoric. Out of 1, 2, 3, and 4, which one is adiabatic?



(*a*) 4 (*b*) 3

(*c*) 2 (*d*) 1

Ans. (*c*): 4 is isobaric process, 1 is isochoric. Out of 3 and 2, 3 has the smaller slope (magnitude) hence is isothermal. Remaining process 2 is adiabatic.

Q12.2. If an average person jogs, he produces 14.5×10^3 cal/min. This is removed by the evaporation of sweat. The amount of sweat evaporated per minute (assuming 1 kg requires 580×10^3 calories for evaporation).

(*a*) 0.025 kg (*b*) 2.25 kg (*c*) 0.05 kg (*d*) 0.20 kg

Ans. (*a*): 580×10^3 calories are needed to convert 1 kg H_2O into steam

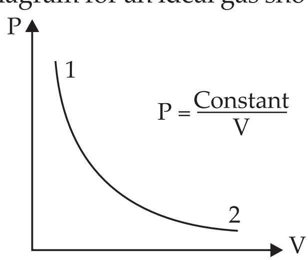
$$\text{1 cal will produce sweat} = \frac{1\text{ kg}}{580 \times 10^3}$$

$$14.5 \times 10^3 \text{ cal will produce sweat} = \frac{14.5 \times 10^3}{580 \times 10^3}\text{ kg}$$

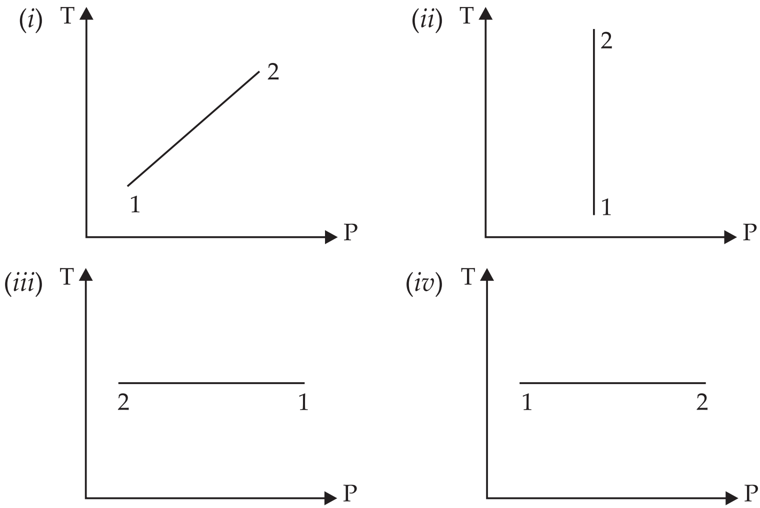
$$= \frac{14.5}{580}\text{ kg per minute}$$

$$= \frac{145}{5800}\text{ kg per minute} = 0.025 \text{ kg per min}$$

Q12.3. Consider P-V diagram for an ideal gas shown in the given figure.



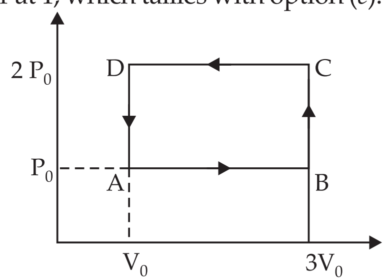
Out of the following diagrams (figure ahead), which represents the T-P diagram?



(*a*) (*iv*) (*b*) (*ii*) (*c*) (*iii*) (*d*) (*i*)

Ans. (*c*): According to P-V diagram at constant temperature, P increases as V decreases. So, it is Boyle's law in options (*iii*) and (*iv*). If P increases at constant temperature, volume V decreases. As in (iii) T-P diagram, P is smaller at 2 and larger at 1, which tallies with option (*c*).

Q12.4. An ideal gas undergoes cyclic process ABCDA as shown in the given PV diagram. The amount of work done by the gas is:



(*a*) 6 P_0V_0 (*b*) $-2P_0V_0$

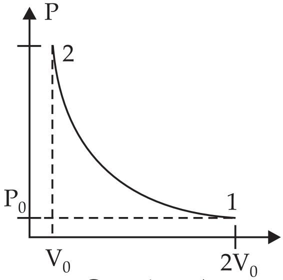
(*c*) $+2P_0V_0$ (*d*) $+4P_0V_0$

Ans. (*d*): The direction of arrows is anticlockwise so work done is negative equal to the area of loop $= -(3V_0 - V_0)(2P_0 - P_0) = -2P_0V_0$ verifies the option (*b*). New work implies external work is done on the system.

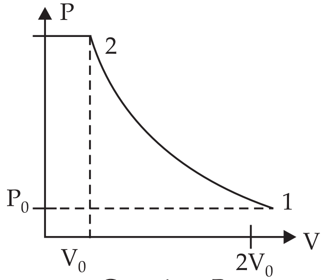
Q12.5. Consider two containers A and B containing identical gases at the same pressure, volume and temperature. The gas in container A is compressed to half of its original volume isothermally while the gas in container B is compressed to half of its original value adiabatically. The ratio of final pressure of gas in B to that of gas in A is:

(*a*) $2^{\gamma-1}$ (*b*) $\left(\frac{1}{2}\right)^{\gamma-1}$ (*c*) $\left(\frac{1}{1-\gamma}\right)^2$ (*d*) $-\left(\frac{1}{\gamma-1}\right)^2$

Ans. (*a*): Consider P-V diagram for container A and B. In both processes compression of gas involve. For isothermal process (gas A) during $(1 \rightarrow 2)$



Container A
(isothermal)



Container B
(adiabatic)

$$P_1V_1 = P_2V_2$$
$$P_0(2V_0) = P_2(V_0)$$
$$P_2 = 2P_0$$

For adiabatic process (gas B) during $(1 \rightarrow 2)$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$
$$P_0(2V_0)^{\gamma} = P_2(V_0)^{\gamma}$$
$$P_2 = \left(\frac{2V_0}{V_0}\right)^{\gamma} P_0 = 2^{\gamma}P_0$$
$$\frac{(P_2)_B}{(P_1)_A} = \frac{2^{\gamma}P_0}{2P_0} = 2^{\gamma-1}$$

Hence verifies the option (*a*).

Q12.6. Three copper blocks of masses M_1, M_2 and M_3 kg respectively are brought into thermal contact till they reach equilibrium. Before contact, they were at T_1, T_2, T_3 $(T_1 > T_2 > T_3)$. Assuming there is no heat loss to the surroundings, the equilibrium temperature T is (*s* is specific heat of copper).

(*a*) $T = \dfrac{T_1 + T_2 + T_3}{3}$

(*b*) $T = \dfrac{M_1T_1 + M_2T_2 + M_3T_3}{M_1 + M_2 + M_3}$

(*c*) $T = \dfrac{M_1T_1 + M_2T_2 + M_3T_3}{3(M_1 + M_2 + M_3)}$

(*d*) $T = \dfrac{M_1T_1s + M_2T_2s + M_3T_3s}{M_1 + M_2 + M_3}$

Ans. (*b*): Let the equilibrium temperature of the system = T

Let $T_1\ T_2 < T < T_3$

As there is no loss to the surroundings.

$$\text{heat lost by } M_3 = \text{Heat gain by } M_1 + \text{Heat gain by } M_2$$
$$M_3s(T_3 - T) = M_1s(T - T_1) + M_2s(T - T_2)$$
$$M_3sT_3 - M_3sT = M_1sT - M_1sT_1 + M_2sT - M_2sT_2$$
$$T(M_3 + M_1 + M_2) = [M_3T_3 + M_1T_1 + M_2T_2]$$

$$T = \frac{M_1T_1 + M_2T_2 + M_3T_3}{M_1 + M_2 + M_3}$$

Hence verifies option (*b*).

MULTIPLE CHOICE QUESTIONS-II

Q12.7. Which of the processes described below are irreversible?

(*a*) The increase in temperature of an iron rod by hammering it.

(*b*) A gas in a small container at a temperature T_1 is brought in contact with a big reservoir at a higher temperature T_2 which increases the temperature of the gas.

(*c*) A quasi-static isothermal expansion of an ideal gas in cylinder fitted with a frictionless piston.

(*d*) An ideal gas is enclosed in a piston cylinder arrangement with adiabatic walls. A weight W is added to the piston, resulting in compression of gas.

Ans. (*a, b, d*)

(*a*) During hammering the rod work is done on rod in hammering, this work converts into heat, raises the temperature of rod, this heat energy cannot be converted into work so process is not reversible.

(*b*) The heat of bigger container transfers to smaller container, till the temperature of both become equal which is average of both. Now, heat from smaller container cannot flow to larger as heat flows from higher temperature to lower.

(*d*) When weight is added to piston then pressure is increased volume decreased it cannot be reversed back itself.

Q12.8. An ideal gas undergoes isothermal process from some initial state *i* to final state *f*. Choose the correct alternatives.

(*a*) $dU = 0$ (*b*) $dQ = 0$

(*c*) $dQ = dU$ (*d*) $dQ = dW$

Ans. (*a, d*): As process is isothermal $\Delta T = 0$ or T constant for an ideal gas dU = change in internal energy $dU = nC_v dT$

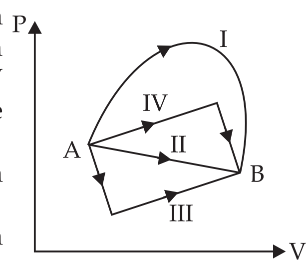
as $dT = 0 \quad \therefore \quad dU = 0$

n = number of moles

$$dQ = dU + dW \quad \Rightarrow \quad dQ = dW$$

verifies option (*a*) and (*d*).

Q12.9. Figure shows the P-V diagram of an ideal gas undergoing a change of state from A to B. Four different parts I, II, III and IV as shown in the figure may lead to the same change of state



(*a*) Change in internal energy is same in IV and III cases, but not in I and II.

(*b*) Change in internal energy in same in all the four cases (*c*) W.D is maximum in case I

(*d*) Work done is minimum in case II.

Ans. (*b, c*): **Main concept used:** *d*U does not depend on P-V path, it depends on initial and final position. WD is P–V is equal to area enclosed with V-axis.

The initial and final position are same for different parts I, II, III, IV. So ΔU is same. Hence option (*a*) rejected verifies option (*b*). As the area enclosed by path I is maximum with V-axis, so W.D. during path I is maximum and minimum is in III.

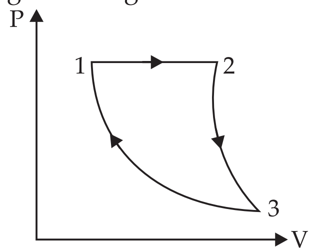
Hence option (*c*) verifies and option *d* rejected.

Q12.10. Consider a cycle followed by an engine see figure.

1 to 2 is isothermal

2 to 3 is adiabatic

3 to 1 is adiabatic



Such a process does not exist because

(*a*) Heat is completely converted to mechanical energy in such a process, which is not possible.

(*b*) Mechanical energy is completely converted to heat in this process, which is not possible.

(*c*) Curves representing two adiabatic processes don't intersect.

(*d*) Curves representing an adiabatic process and an isothermal process don't intersect.

Ans. (*a, c*): (*a*) The given process is cyclic which starts from 1 and ends up at 1 again rd

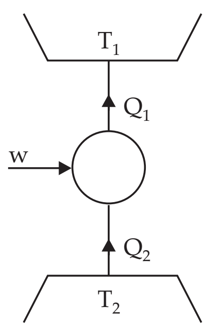
so $dU = 0$ i.e. $dQ = dU + dW$

$\Rightarrow$ $dQ = dW$

Hence heat energy supply to system converts totally into mechanical work which is not possible by second law of thermodynamics verifies option (*a*).

(*c*) P-V curve 2 to 3 and 3 to 1 both are adiabatic it cannot be possible without supply energy hence process 3 to 1 cannot be adiabatic. Verifies option (*c*) and rejects option (*d*).

Q12.11. Consider a heat engine as shown in figure, Q_1 and Q_2 are heat added to heat bath T_1 and heat taken from T_2 in one cycle of engine. W is mechanical work done on the engine.



If W > 0, then possibilities are:

(*a*) $Q_1 > Q_2 > 0$ (*b*) $Q_2 > Q_1 > 0$

(*c*) $Q_2 < Q_1 < 0$ (*d*) $Q_1 < 0$ and $Q_2 > 0$

Ans. (*a, c*): From fig $Q_1 = W + Q_2$

$\therefore$ $W > 0$ So $\because Q_1 - Q_2 > 0$ or $Q_1 > 0$

$\therefore$ $Q_1 > Q_2 > 0$ if both Q_1, Q_2 positive verifies option (*a*).

or $Q_2 < Q_1 < 0$ if both Q_1 Q_2 negative verifies option (*c*).

VERY SHORT ANSWER QUESTIONS

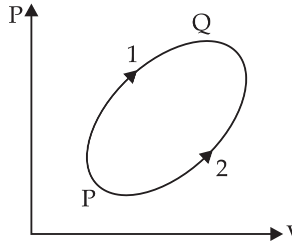
Q12.12. Can a system be heated and its temperature remains constant?

Ans. It is given that $\Delta T = 0 \quad \Rightarrow \quad \Delta U = 0$

$\therefore \quad \Delta Q = \Delta U + \Delta W$

$\Rightarrow \quad \Delta Q = \Delta W$ So heat supplied to the system is utilized in expansion system is isothermal.

Q12.13. A system goes from P to Q by two different paths in P-V diagram as shown in figure. Heat given to the system in path 1 is 1000 J. The work done by the system along path 1 is more than path 2 by 100 J. What is the heat exchanged by the system in path 2?



(given)

Ans. For path (1) $Q_1 = +1000$ J

$$W.D. = W_1 - W_2 = 100$$

$$W_1 = \text{WD through path 1}$$

$$W_2 = \text{WD through path 2}$$

$$\therefore \quad W_2 = W_1 - 100$$

As change in internal energy by path 1 and 2 are same

$$\Delta U = Q_1 - W_1 = Q_2 - W_2$$

$$1000 - W_1 = Q_2 - (W_1 - 100)$$

$$1000 = Q_2 + 100$$

$$Q_2 = 900 \text{ J.}$$

Q12.14. If a refrigerator's door is kept open, will the room become cool or hot? Explain.

Ans. If a refrigerator door is kept open the room will become hotter, because amount of heat absorbed from inside the refrigerator and work done on refrigerator by electricity both will be rejected by refrigerator in room.

Q12.15. Is it possible to increase the temperature of a gas without adding heat to it? Explain.

Ans. During adiabatic compression the temperature of gas increase while no heat is given to system

In adiabatic compression $dQ = 0$

$$dQ = dU + dW$$

$$\therefore \quad dU = -dW$$

So in compression work is done on system (WD (–)ve). So dU +ve and increase the temperature of system.

So as internal energy of gas (ideal) increases its temperature increases.

Q12.16. Air pressure in a car tyre increases during driving. Explain.

Ans. During driving, reaction force due to force on tyres, temperature of gas increases so gas inside tyres expands as volume inside the tyre remains constant (Charle's law) so temperature of car tyre increases during driving (as $P \propto T$).

SHORT ANSWER TYPE QUESTIONS

Q12.17. Consider a Carnot's cycle operating between $T_1 = 500$ K and $T_2 = 300$ K producing 1 kJ of mechanical work per cycle. Find the heat transferred to the engine by the reservoirs.

Ans. Efficiency of Carnot's engine $\eta = 1 - \frac{T_2}{T_1}$

Temperature of source or reservoir $= T_1 = 500$ K
Temperature of sink $= T_2 = 300$ K

$$\therefore \quad \eta = 1 - \frac{T_2}{T_1}$$

$$\frac{\text{Output work}}{\text{Input work (E)}} = 1 - \frac{300}{500}$$

$$\frac{1000 \text{ J}}{x} = 1 - 0.6$$

$$\frac{1000}{x} = 0.4$$

$$x = \frac{1000}{0.4} = 2500 \text{ J}.$$

Q12.18. A person of mass 60 kg wants to lose 5 kg by going up and down a 10 m high stairs. Assume he burns twice as much fat while going up than coming down. If 1 kg of fat is brunt on expending 7000 k cal, how many times must he go up and down to reduce his weight by 5 kg?

Ans. Energy produced by 1 kg fat = 7000 k cal

$$\text{Energy produced by 5 kg fat} = 5 \times 7000 \text{ k cal} = 35000 \text{ k cal.}$$

$$= 35 \times 10^6 \text{ cal}$$

$$\text{Energy consumed to go up one time} = mgh$$

$$\text{Energy consumed to come down one time} = \frac{1}{2} mgh \quad \text{(given)}$$

$$\therefore \text{ Energy consumed to go up and down one time} = mgh + \frac{1}{2} mgh$$

$$E = \frac{3}{2} mgh = \frac{3}{2} \times 60 \times 10 \times 10 = 9000 \text{ J}$$

$$\text{Energy consume 1 time} = \frac{9000}{4.2} \text{ cal.}$$

Let he go up and down n time to consumed energy 35×10^6 cal

$$n \times \frac{9000}{4.2} = 35 \times 10^6$$

$$n = \frac{35 \times 1000 \times 1000 \times 42}{9000 \times 10} = \frac{3500}{3} \times 14$$

$$= \frac{49000}{3} = 16.3 \times 10^3 \text{ times.}$$

Q12.19. Consider a cycle tyre being filled with air by a pump. Let V be the volume of tyre (fixed) and at each stroke of the pump $\Delta V[<<V]$ of air is transferred to the tube adiabatically. What is the work done when the pressure in the tube is increased from P_1 to P_2?

Ans. Air is transferred into tyre adiabatically let initial volume of air in tyre V and after pumping one stroke it become (V + dV) and pressure increased from P to (P + *d*P) then

$$P_1V_1^\gamma = P_2V_2^\gamma$$
$$P(V + dV)^\gamma = (P + dP)V^\gamma$$
$$PV^\gamma\left[1 + \frac{dV}{V}\right]^\gamma = P\left[1 + \frac{dP}{P}\right]V^\gamma$$

as volume of tyre V remains constant

$$PV^\gamma\left[1 + \gamma\frac{dV}{V}\right] = PV^\gamma\left[1 + \frac{dP}{P}\right]$$

[on expanding by binomial theorem neglecting the higher terms of ΔV as $\Delta V << V$]

$$1 + \gamma\frac{dV}{V} = 1 + \frac{dP}{P}$$
$$dV = \frac{VdP}{\gamma P}$$

Integrating both side in limits W_1 to W_2 and $P_1 \to P_2$

$$\int PdV = \int_{P_1}^{P_2}\frac{VdP}{\gamma}$$
$$\int_{W_1}^{W_2} dW = \frac{V}{\gamma}(P_2 - P_1) \qquad \text{(V = constant)}$$
$$W = \frac{(P_2 - P_1)V}{\gamma}$$

Q12.20. In a refrigerator one removes heat from lower temperature and deposits to the surroundings at a higher temperature. In this process, mechanical work has to be done, which is provided by an electric motor. If the motor of 1 kW power, and heat is transferred – 3°C to 27°C, find the heat taken out of the refrigerator per second assuming its efficiency is 50% of a perfect engine.

Ans. Carnot's engine is perfect heat engine operating between two temperature T_1 and T_2 (source and sink). Refrigerator is also Carnot's engine working in reverse order its efficiency is η

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{273 - 3}{273 + 27} = 1 - \frac{270}{300} = 1 - .9 = .1 = \frac{1}{10}$$

Efficiency of refrigerator's 50% of perfect engine

$\therefore$ Efficiency of refrigerator = 50% of 1 = .5

Net efficiency = $\eta' = 0.5 \times 0.1 = 0.05$

$\therefore$ Coefficient of performance $\beta = \dfrac{Q_2}{W} = \dfrac{1-\eta'}{\eta'}$

$$\beta = \frac{1-0.05}{0.05} = \frac{0.95}{0.05} = 19$$

Q_2 = 19% W.D. by motor on refrigerator

$= 19 \times 1$ kW = 19 kJ/s

Q12.21. If the coefficient of performance of a refrigerator is 5 and operates at the room temperature (27°C), find the temperature inside the refrigerator.

Ans. We know,

$$\beta = \frac{T_2}{T_1 - T_2} \qquad \begin{pmatrix} \beta = 5 \\ T_1 = 27 + 273 = 300\text{ K} \end{pmatrix}$$

$$5 = \frac{T_2}{300 - T_2} \Rightarrow T_2 = 1500 - 5T_2$$

$$T_2 + 5T_2 = 1500 \Rightarrow 6\,T_2 = 1500$$

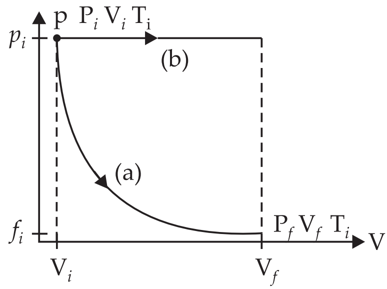
$$T_2 = \frac{1500}{6} = 250\text{ K} = 250 - 273 = -23°\text{C}.$$

Q12.22. The initial state of a certain gas is $(P_iV_iT_i)$. It undergoes expansion till its volume become V_f. Consider the following two cases.

(*a*) The expansion takes place at constant temperature.

(*b*) The expansion takes place at constant pressure.

Plot P–V diagram for each case. In which of the two cases, is the work done by the gas more?



Ans. (*a*) The expansion from V_i to V_f temperature T_i remains constant so isothermal expansion i.e. $P_iV_i = P_fV_f$ constant T.

(*b*) The expansion is at constant pressure p_i, so isobaric process so graph P-V will be parallel to V axis till its volume becomes V_f. As the area enclosed by graph (*a*) is less than (*b*) with volume axis so W.D. by process (*b*) is more than of (*a*).

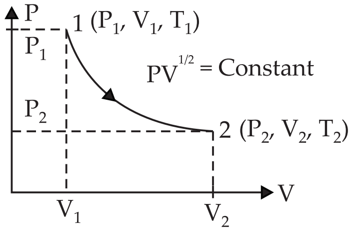
LONG ANSWER TYPE QUESTIONS

Q12.23. Consider a P-V diagram in which the path followed by one mole of perfect gas in a cylindrical container is shown in figure.

(*a*) Find work done when the gas is taken from state (1) to state (2).

(*b*) What is the ratio of temperature T_1/T_2, if $V_2 = 2V_1$?

(*c*) Given the internal energy for one mole of gas at temperature T is (3/2) RT, find the heat supplied to the gas when it is taken from state (1) to (2) with $V_2 = 2V_1$.



Ans. $\because PV^{1/2} = \text{constant} = K(\text{given})$ or $P_1V_1^{1/2} = P_2V_2^{1/2} = K$ and $P = K/V^{1/2}$

(*a*) Work done for process from 1 to 2

$$WD = \int_{V_1}^{V_2} P\cdot dV = \int_{V_1}^{V_2} \frac{K}{V^{1/2}} dV = K\int_{V_1}^{V_2} V^{-(1/2)}dV$$

$$WD = K\left[\frac{V^{1/2}}{\frac{1}{2}}\right]_{V_1}^{V_2} = 2K\left[\sqrt{V_2} - \sqrt{V_1}\right]$$

WD from V_1 to V_2, i.e., $dW = 2P_1V_1^{1/2}\left[\sqrt{V_2} - \sqrt{V_1}\right]$

$$= 2P_2V_2^{1/2}\left[\sqrt{V_2} - \sqrt{V_1}\right]$$

(*b*) from gas equation of ideal gas $PV = nRT$

$$\Rightarrow \quad T = \frac{PV}{nR} = \frac{P\sqrt{V}\sqrt{V}}{nR} = \frac{K\sqrt{V}}{nR}$$

$$T_1 = \frac{K\sqrt{V_1}}{nR} \quad \text{and} \quad T_2 = \frac{K\sqrt{V_2}}{nR}$$

$$\frac{T_1}{T_2} = \frac{\frac{K\sqrt{V_1}}{nR}}{\frac{K\sqrt{V_2}}{nR}} = \frac{\sqrt{V_1}}{\sqrt{V_2}} = \sqrt{\frac{V_1}{2V_1}} \quad (\because \; V_2 = 2V_1 \text{ given})$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{1}{\sqrt{2}} \qquad \ldots(ii)$$

required ratio is $1:\sqrt{2}$.

(c) Given that internal energy U of gas is

$$U = \left(\frac{3}{2}\right)RT$$

$$\Delta U = \frac{3}{2}R\,dT = \frac{3}{2}R(T_2 - T_1)$$

$$\because \quad T_2 = \sqrt{2}\,T_1, \text{ from part } (b)$$

$$\Delta U = \frac{3}{2}R\left[\sqrt{2}T_1 - T_1\right] = \frac{3}{2}RT_1(\sqrt{2} - 1)$$

$$\text{from part } (a)\, dW = 2P_1V^{1/2}\left(\sqrt{V_2} - \sqrt{V_1}\right)$$

$$\because \quad V_2 = 2V_1 \quad \text{(given)}$$

$$\text{so} \quad \sqrt{V_2} = \sqrt{2}\sqrt{V_1} \quad \text{then}$$

$$dW = 2P_1V_1^{1/2}\left(\sqrt{2}\sqrt{V_1} - \sqrt{V_1}\right)$$

$$= 2P_1V_1^{1/2}\sqrt{V_1}[\sqrt{2} - 1]$$

$$dW = 2P_1V_1(\sqrt{2} - 1)$$

$$dW = 2nRT_1(\sqrt{2} - 1) \qquad (\because \quad P_1V_1 = nRT_1)$$

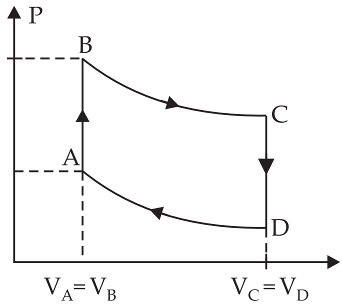
$$\because \quad n = 1 \quad \therefore \quad dW = 2RT_1(\sqrt{2} - 1)$$

$$\therefore \quad dQ = dW + dU = 2RT_1(\sqrt{2} - 1) + \frac{3}{2}RT_1(\sqrt{2} - 1)$$

$$= (\sqrt{2} - 1)RT_1\left[2 + \frac{3}{2}\right]$$

$$dQ = -(\sqrt{2} \quad 1)RT \ .$$

Q12.24. A cycle followed by an engine [made of one mole of perfect gas in a cylinder with piston] is shown in figure.



A to B : constant volume
B to C : adiabatic
C to D : volume constant
D to A : adiabatic

$$V_C = V_D = 2V_A = 2V_B$$

(*a*) In which part of the cycle heat is supplied to the engine from outside?

(*b*) In which part of the cycle heat is being given to the surrounding by the engine?

(*c*) What is the work done by the engine in one cycle? Write your answer in term of P_A, P_B, V_A?

(*d*) What is the efficiency of the engine?

$$\left[\gamma = \frac{5}{3} \text{ for the gas}\right], \quad \left[C_V = \frac{3}{2}R \text{ for one mole}\right]$$

Ans. (*a*): Heat is supplied to engine in part AB in which

$dV = 0$ so $\because$ $dW = P \cdot dV$ $\therefore$ $dW = P \cdot 0$ $\therefore$ $dW = 0$

By Ist law of thermodynamics $dQ = dU + dW$

$$dQ = dU$$

i.e. heat energy supplied to system does not work, but increase the internal energy dU of gas or system

$$\therefore \quad P = \frac{nRT}{V} \quad \because \quad V = \text{constant} \quad \therefore \quad P \propto T$$

as pressure is increased at constant volume it temperature i.e. internal energy increased (*b*) Heat is given out by system in part CD. (inverse of AB). In part CD, $dV = 0$, pressure decreases so temperature also decreases ($P \propto T$) i.e. energy given out by the system to surrounding.

(c) WD by system = $\int_A^B P\,dV + \int_B^C P\,dV + \int_C^D P\,dV + \int_D^A P\,dV$

in A to B and C to D $dV = 0$

$$\int_A^B P\,dV = 0 \quad \text{and} \quad \int_C^D P\,dV = 0$$

for adiabatic change $PV^\gamma = K$ ...(*i*)

$$P = \frac{K}{V^\gamma}$$

$$\int_B^C P\,dV = \int_{V_B}^{V_C} \frac{K}{V^\gamma}\,dV = K\int_{V_B}^{V_C} V^{-\gamma}\,dV = K\left[\frac{V^{-\gamma+1}}{1-\gamma}\right]_{V_B}^{V_C}$$

$$\int_B^C P\,dV = \frac{K}{1-\gamma}\left[V_C^{1-\gamma} - V_B^{1-\gamma}\right] \quad ...(ii)$$

Similarly D to A is also adiabatic so

$$\int_D^A P\,dV = K\frac{(V_A^{1-\gamma} - V_D^{1-\gamma})}{1-\gamma} = \frac{K}{1-\gamma}\left[V_A^{1-\gamma} - V_D^{1-\gamma}\right]$$

$$= \frac{K\,V_A^{1-\gamma} - KV_D^{1-\gamma}}{1-\gamma}$$

For adiabatic

$$\int_D^A P\,dV = \frac{P_A V_A^\gamma V_A^{1-\gamma} - P_D V_D^\gamma V_D^{1-\gamma}}{1-\gamma} \qquad [\because\ PV^\gamma = K \text{ from } (i)]$$

$$\int_D^A P\,dV = \frac{P_A V_A - P_D V_D}{1-\gamma}$$

Similarly

$$\int_B^C P\,dV = \frac{P_C V_C - P_B V_B}{1-\gamma}$$

$$\therefore \quad \text{Total} \quad WD = 0 + \frac{P_C V_C - P_B V_B}{1-\gamma} + 0 + \frac{P_A V_A - P_D V_D}{1-\gamma}$$

$$= \frac{1}{1-\gamma}[P_C V_C - P_B V_B + P_A V_A - P_D V_D]$$

For adiabatic change $P_B V_B^\gamma = P_C V_C^\gamma$

$$P_C = \frac{P_B V_B^\gamma}{V_C^\gamma} = P_B\left(\frac{V_B}{V_C}\right)^\gamma = P_B\left(\frac{V_B}{2V_B}\right)^\gamma$$

$$P_C = P_B \frac{1}{2^\gamma} = P_B 2^{-\gamma} \quad \text{similarly } P_D = P_A 2^{-\gamma}$$

$$\therefore \quad \text{Net WD} = \frac{1}{1-\gamma}[P_B V_C 2^{-\gamma} - P_B V_B + P_A V_A - P_A V_D 2^{-\gamma}]$$

$$= \frac{1}{1-\gamma}[P_B 2V_B 2^{-\gamma} - P_B V_B + P_A V_A - P_A 2V_A 2^{-\gamma}]$$

$$= \frac{1}{1-\gamma}[-P_B V_B[-2^{1-\gamma} + 1] + P_A V_A [1 - 2^{1-\gamma}]]$$

$$= \frac{1}{1-\gamma}[-2^{1-\gamma} + 1][-P_B V_B + P_A V_A]$$

$$= \frac{1}{1-\gamma}(-2^{1-\gamma} + 1)[-P_B V_A + P_A V_A]$$

$$\because 2V_B = 2V_A \quad \Rightarrow \quad V_A = V_B$$

$$= \frac{1}{1-\frac{5}{3}}[-2^{1-5/3} + 1][-P_B + P_A]V_A$$

$$= +\frac{3}{2}(-2^{-2/3} + 1)[P_B - P_A]V_A$$

$$= +\frac{3}{2}\left[1 - \left(\frac{1}{2}\right)^{2/3}\right][P_B - P_A]V_A .$$

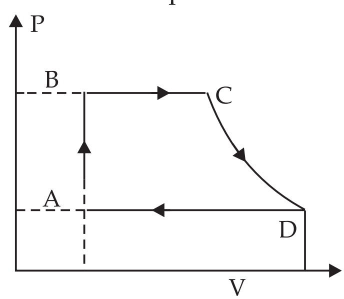
Q12.25. A cycle followed by an engine (made of one mole of an ideal gas in a cylinder with a piston) is shown in figure. Find heat exchanged by the engine, with the surroundings for each section of the cycle. $\left(C_V = \frac{3}{2}R\right)$.

(*a*) AB: constant volume

(*b*) BC: constant pressure

(*c*) CD: adiabatic

(*d*) DA: constant pressure



Ans. (*a*): For A → B, $dV = 0$

so $$dW = \int P \cdot dV = \int P \times 0 = 0$$

$$dW = 0$$

By 1st law of thermodynamics

$$dQ = dU + dW = dU + 0$$

$$\therefore \quad dQ = dU$$

$$(\because dQ = nC_V dT)$$

$$n = 1; \quad C_V = \frac{3}{2}R$$

so $$dQ = 1\frac{3}{2}R(T_B - T_A) \qquad ...(i)$$

$$dU = dQ = \frac{3}{2}(RT_B - RT_A) = \frac{3}{2}(P_B V_B - P_A V_A)$$

∴ Heat exchange [to system]

$$dQ_1 = dU = \frac{3}{2}(P_B V_B - P_A V_A)$$

(*b*) For B to C, $\Delta P = 0 \quad n = 1$

$$dQ = dU + dW = C_V(dT) + P_B\,dV$$

$$dQ_2 = \frac{3}{2}R(T_C - T_B) + P_B(V_C - V_B)$$

$$= \frac{3}{2}(T_CR - RT_B) + P_BV_C - P_BV_B$$

$$= \frac{3}{2}[P_CV_C] - \frac{3}{2}[P_BV_B] - P_BV_B + P_BV_C$$

$$V_A = V_B \quad \text{and} \quad P_B = P_C$$

$$\therefore \quad dQ_2 = \frac{3}{2}P_BV_C - \frac{3}{2}P_BV_A - P_BV_A + P_BV_C$$

$$= \frac{5}{2}P_BV_C - \frac{5}{2}P_BV_A$$

$$dQ_2 = \frac{5}{2}P_B[V_C - V_A].$$

(*c*) For diagram C → D, adiabatic change

$\therefore \quad dQ_3 = 0$ (No exchange of heat)

(*d*) For diagram D → A, $\Delta P = 0$ Compression of gas from volume V_D to V_A at constant pressure hence heat exchange similar to part (*b*) i.e. Heat exchange $dQ_3 = \frac{5}{2}P_A(V_A - V_D)$

Q12.26. Consider that an ideal gas (*n* moles) is expanding in a process given by $P = f(V)$, which passes through a point (V_0, P_0). Show that the gas is absorbing heat at (P_0, V_0) if the slope of the curve $P = f(V)$ is larger than the slope of the adiabatic passing through (P_0, V_0).

Ans. Slope of graph at $(V_0, P_0) = \left(\frac{dP}{dV}\right)_{(V_0P_0)}$

$P = f(V)$ for adiabatic process PV^γ = constant (K)

or $\quad P = \frac{K}{V^\gamma}$ or $\frac{dP}{dV} = K(-\gamma)\,V^{-\gamma-1}$

$$\frac{dP}{dV} = -\gamma PV^\gamma V^{-\gamma}V^{-1} = -\frac{\gamma P}{V}$$

$\left(\frac{dP}{dV}\right)_{(P_0, V_0)} = \frac{-\gamma P_0}{V_0}$ Heat absorbed by in the process $P = f(V)$

$$dQ = dU + dW$$

$$dQ = nC_V dT + P dV \quad \ldots(i)$$

$$PV = nRT$$

$$T = \frac{PV}{nR} = \frac{V}{nR}f(V)$$

$$\frac{dT}{dV} = \frac{1}{nR}[f(V) + Vf'(V)]$$

$$\frac{dQ}{dV} = nC_V \frac{dT}{dV} + P \cdot \frac{dV}{dV} = \frac{nC_V}{nR}[f(V) + Vf'(V)] + P$$

$$\left(\frac{dQ}{dV}\right)_{V=V_0} = \frac{C_V}{R}[f(V_0) + V_0\, f'(V_0)] + f(V_0) \quad [\because\ P = f(V) \text{ given}]$$

$$= f(V_0)\left[\frac{C_V}{R} + 1\right] + V_0 f'(V_0)\frac{C_V}{R}$$

$$C_p - C_V = R \quad \Rightarrow \quad \frac{C_p}{C_V} - 1 = \frac{R}{C_V}$$

$$\therefore \quad \gamma - 1 = \frac{R}{C_V} \quad \Rightarrow \quad C_V = \frac{R}{\gamma - 1} \quad \Rightarrow \quad \frac{C_V}{R} = \frac{1}{\gamma - 1}$$

$$\left(\frac{dQ}{dV}\right)_{V=V_0} = f(V_0)\left[\frac{1}{\gamma - 1} + 1\right] + V_0 f'(V_0)\frac{1}{\gamma - 1}$$

$$= f(V_0)\left[\frac{1 + \gamma - 1}{\gamma - 1}\right] + \frac{V_0 f'(V_0)}{\gamma - 1}$$

$$= \frac{\gamma}{(\gamma - 1)} f(V_0) + V_0 \frac{f'(V_0)}{(\gamma - 1)}$$

$$= \frac{1}{(\gamma - 1)}[\gamma f(V_0) + V_0 f'(V_0)] \qquad (\because\ f(V_0) = P_0)$$

$$\left(\frac{dQ}{dV}\right)_{V=V_0} = \frac{1}{(r - 1)}[\gamma P_0 + V_0 f'(V_0)]$$

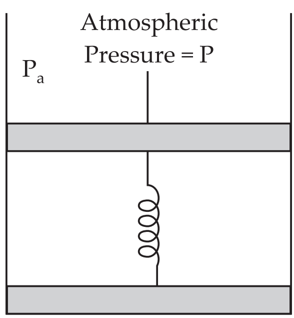
$$\therefore \quad \left(\frac{dQ}{dV}\right)_{V=V_0} > 1 \quad \therefore \quad \text{and} \quad \gamma > 1 \quad \text{so} \quad \frac{1}{\gamma - 1} \text{ is +ve}$$

$$\therefore \ \gamma P_0 + V_0 f'(V_0) > 0$$

$$V_0 f'(V_0) > -\gamma P_0$$

$$f'(V_0) > \frac{-\gamma P_0}{V_0}.$$

Q12.27. Consider one mole of perfect gas in a cylinder of unit cross-section with a piston attached (figure). A spring (spring constant k) is attached (unstretched length L) to the piston and to the bottom of the cylinder. Initially the spring is unstretched and the gas is in equilibrium. A certain amount of heat Q is supplied to the gas causing an increase of volume from V_0 to V_1.



(*a*) What is the initial pressure of the system?

(*b*) What is the final pressure of the system?

(*c*) Using the first law of thermo-dynamics, write down a relation between Q, P_a, V_1, V_0 and k.

Ans. (*a*): It is considered that piston is mass less and piston is balanced by atmospheric pressure (P_a). So the initial pressure of system inside the cylinder = P_a.

(*b*) On supply heat Q. Volume of gas increases from V_0 to V_1 and spring stretched also.

So increase in volume = $V_1 - V_0$

If displacement of piston is x then volume increase in cylinder

$$= \text{Area of base} \times \text{height} = A \times x$$

$$A \times x = V_1 - V_0 \qquad (A = \text{area of cross section of cylinder})$$

$$\therefore \qquad x = \frac{V_1 - V_0}{A}$$

$$\text{Force exerted by spring } F_S = Kx = \frac{K(V_1 - V_0)}{A}$$

as the piston is of unit area of cross-section $\therefore$ A = 1

Force due to spring = $K(V_1 - V_0)$ on unit area can be say press due to spring = $K(V_1 - V_0)$

Final total pressure on gas $P_f = P_a + K(V_1 - V_0)$

(*c*) By Ist law of thermodynamics $dQ = dU + dW$

$$dU = C_V(T - T_0)$$

T = final temperature of gas

T_0 = initial temperature of gas

n = 1

$$T_f = T = \frac{P_f V_f}{R} = \frac{[P_a + k(V_1 - V_0)]V_1}{R}$$

W.D. by gas = $p.\, dV$ + increase in PE of spring

$$dW = P_a(V_1 - V_0) + \frac{1}{2}kx^2$$

Now $$dQ = dU + dW$$

$$= C_V\,(T - T_0) + P_a(V_1 - V_0) + \frac{1}{2}kx^2$$

$$dQ = C_V(T - T_0) + P_a(V_1 - V_0) + \frac{1}{2}k\,(V_1 - V_0)^2$$

It is required relation.

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