15 Waves

MULTIPLE CHOICE QUESTIONS-I

Q15.1. Water waves produced by a motor boat sailing in water are

(*a*) neither longitudinal nor transverse
(*b*) both longitudinal and transverse
(*c*) only longitudinal
(*d*) only transverse

Ans. (*b*): As the waves are produce by motor boat on surfaces as well as inside water. So the waves are transverse as well as longitudinal both.

Q15.2. Sound waves of wavelength λ travelling in a medium with a speed of v m/s enter into another medium where its speed is $2v$ m/s wavelength of sound waves in the second medium is:

(*a*) λ (*b*) $\frac{\lambda}{2}$ (*c*) 2λ (*d*) 4λ

Ans. (*c*): When wave passes from one medium to another its frequency (ν) does not change, but its velocity and wavelength changes.

$$v = \nu\lambda \quad \text{or} \quad \nu = \frac{v}{\lambda}$$

$$\frac{v}{\lambda} = \frac{2v}{\lambda_2} \Rightarrow \lambda_2 = 2\lambda.$$ Hence verifies the option (*c*).

Q15.3. Speed of the sound wave in air

(*a*) is independent of temperature
(*b*) increases with pressure
(*c*) increase on increasing humidity
(*d*) decreases with increase in humidity

Ans. (*c*): Speed of sound (longitudinal) wave in air is $v = \sqrt{\frac{\gamma P}{\rho}}$. The density of water vapours is small (rises up) than the air so on increasing humidity the density of medium decrease in turn increases the speed of sound in air by $v \propto \frac{1}{\sqrt{\rho}}$ (relation). Hence verifies the option (*c*).

Q15.4. Change in temperature of the medium changes

(*a*) frequency of sound waves
(*b*) amplitude of sound waves
(*c*) wavelength of sound waves
(*d*) loudness of sound waves

Ans. (*c*): We know that $v_t = v_0 (1 + .61t)$ or $v_t \propto \sqrt{T}$. So on increasing temperature the speed also increases as frequency does not change during propagation of wave by formula $v = \nu\lambda$. So velocity v and wavelength λ both increases. Hence, verifies the option (*c*).

Q15.5. With propagation of longitudinal wave through a medium, the quantity transmitted is

(*a*) a matter
(*b*) energy
(*c*) energy and matter
(*d*) energy matter and momentum

Ans. (*b*): During propagation of any wave in a medium only energy is transmitted from one point to another. Matter does not change its own position it vibrates about its mean position only.

Q15.6. Which of the following statements are true for a wave motion?

(*a*) Mechanical transverse waves can propagate through all medium.
(*b*) Longitudinal wave can propagate through solid only
(*c*) Mechanical transverse wave can propagate through solid only
(*d*) Longitudinal wave can propagate through vacuum.

Ans. (*c*): Mechanical transverse wave can propagate through solid, and on surface of liquid also, so best option is (*c*).

Transverse wave can not propagate in gases rejects option (*a*).

Longitudinal can propagate through gases rejects option (*b*).

Longitudinal waves are not e.m. waves and can mechanical wave can never propagate in vacuum rejects option (*d*).

Q15.7. Sound wave is passing through air column in the form of compression and rarefaction. In consecutive compressions and rarefactions.

(*a*) density remains constant
(*b*) Boyle's law is obeyed
(*c*) bulk modulus of air oscillates
(*d*) there is no transfer of heat

Ans. (*d*): (*i*) The density of medium particles are maximum and minimum at compression and rarefaction points, so rejects option (*a*).

(*ii*) Also density changes very rapidly, so temperature of medium increases. So rejects option (*b*).
(*iii*) Bulk modulus of air remains constant, rejects option (*c*).
(*iv*) The time of compressions and rarefaction is very small so heat does not transfer.

Q15.8. The equation of plane progressive wave is given by $y = 0.6 \sin 2\pi\left[t + \frac{x}{2}\right]$. On reflection from a denser medium its amplitude becomes $\frac{2}{3}$ of the amplitude of the incident wave. The equation of reflected wave is (*a*) $y = 0.6 \sin 2\pi \left[t + \frac{x}{2}\right]$ (*b*) $y = -0.4 \sin 2\pi \left[t + \frac{x}{2}\right]$

(*c*) $y = 0.4 \sin 2\pi \left[t + \frac{x}{2}\right]$ (*d*) $y = -0.4 \sin 2\pi \left[t - \frac{x}{2}\right]$

Ans. (*b*): After reflection of wave changes by phase 180°

$$y_i = 0.6 \sin 2\pi \left[t + \frac{x}{2}\right]$$

$$y_r = \left(\frac{2}{3} \times 0.6\right) \sin 2\pi \left[\pi + t + \frac{x}{2}\right]$$

$$y_r = -0.4 \sin 2\pi \left(t + \frac{x}{2}\right)$$. Verifies the option (*b*).

Q15.9. A string of mass 2.5 kg is under Tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at the one end of the string. The disturbance will reach the other end in

(*a*) one second (*b*) 0.5 second

(*c*) 2 second (*d*) data given insufficient

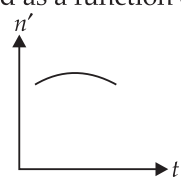
Ans. (*b*): M = mass string = 2.5 kg, $l = 20$ m

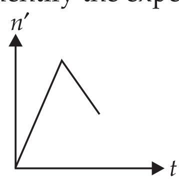
$$m = \text{mass per unit length} = \frac{M}{l} = \frac{2.5}{20} = 0.125 \text{ kg/m}$$

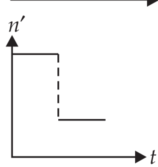
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200}{0.125}} = \sqrt{1600} = 40 \text{ m/s}$$

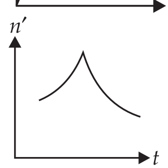
$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{20 \text{ m}}{40 \text{ m/s}} = \frac{1}{2} \text{ sec} = 0.5 \text{ sec}.$$

Q15.10. A train whistling at constant frequency is moving towards a station at a constant speed v. The train goes past a stationary observer on station. The frequency n' of the sound as heard by observer is plotted as a function of time (t) figure. Identify the expected curve.

(*a*)


(*b*)


(*c*)


(*d*)


Ans. (*c*): When observer is at rest and source of sound is moving towards observer then observed frequency n'.

$$n' = \left(\frac{v}{v - v_s}\right) n_o$$

where n_o = original frequency of source of sound

v = speed of sound in medium

$\therefore n' > n_o$ $\quad v_s$ = speed of source

When source is moving away from observer

$$n'' = \frac{v}{(v + v_s)} n_o \qquad n'' < n_o$$

Hence, the frequencies in both cases are same and $n' > n''$. So graph (c) verifies the answer.

MULTIPLE CHOICE QUESTIONS-II MORE THAN ONE OPTION

Q15.11. A transverse harmonic wave on a string is described by $y(x, t) = 3.0 \sin\left(36t + 0.018x + \frac{\pi}{4}\right)$, where x, y are in cm and t is in second. The positive direction of x is from left to right.

(*a*) The wave is travelling right to left

(*b*) The speed of the wave is 20 m/s

(*c*) Frequency of the wave is 5.7 Hz.

(*d*) The least distance between the two successive crests in the wave is 2.5 cm.

Ans. (*a, b, c*): The standard form of a wave propagated from left to right i.e., in +ve direction

$$y = a \sin(\omega t - kx + \phi) \text{ and}$$

$$y = 3.0 \sin\left(36t + 0.018x + \frac{\pi}{4}\right) \qquad \text{(given)}$$

(*a*) As in given equation x is in positive sign so given wave travelling from right to left. Verifies option (*a*).

(*b*),(*c*) $\omega = 36$ or $2\pi n = 36$ or $\nu = \frac{36}{2\pi} = \frac{18}{3.14} = 5.7$ Hz

Verifies option (*c*) $n = 5.7$ Hz

$k = 0.018 \quad \frac{2\pi}{\lambda} = 0.018 \Rightarrow \lambda = \frac{2\pi}{0.018}$

$$\therefore \quad v = \nu\lambda = \frac{18}{\pi} \times \frac{2\pi}{0.018} = 2000 \text{ cm/s} = 20 \text{ m/s}$$

Verifies the option (*b*)

(*d*) Distance between two successive crests $= \lambda = \frac{2\pi}{0.018}$

$$\lambda = \frac{\pi}{0.009} = \frac{3.14 \times 1000}{9} = \frac{3140}{9} \text{ cm}$$

$$\lambda = 348.8 \text{ cm} = 3.48 \text{ m} \neq 2.5 \text{ cm}$$

Hence, rejects the option (*d*).

Q15.12. The displacement of a string is given by

$$y(x, t) = 0.06 \sin \frac{2\pi x}{3} \cos 120\pi t,$$

where x, y are in m and t in s. The length of the string is 1.5 m and its mass is 3.0×10^{-2} kg.

(*a*) It represents the progressive wave of frequency 60 Hz
(*b*) It represents the stationary wave of frequency 60 Hz
(*c*) It is the result of superposition of two waves of wavelength 3 m, frequency 60 Hz each travelling with a speed of 180 m/s in opposite direction
(*d*) Amplitude of this wave is constant.

Ans. (*b, c*): We know that standard equation of stationary wave is

$$y(x, t) = a \sin (kx) \cos (\omega t)$$

and given equation is $y(x, t) = 0.06 \sin \left(\frac{2\pi}{3} x\right) \cos [(120\pi)t]$

(*a*) Clearly the given wave is stationary. Hence rejects the option (*a*).
(*b*) Comparing both equation $\omega = 120\pi$

$$2\pi\nu = 120\pi \quad \text{or} \quad \nu = \frac{120}{2} = 60 \text{ Hz}$$

Verifies the option (*b*).

(*c*) $\frac{2\pi}{\lambda} = k$ from eqn., $k = \frac{2\pi}{3}$

$$\therefore \frac{2\pi}{\lambda} = \frac{2\pi}{3}$$

$\Rightarrow \lambda = 3$ m, $\nu = 60$ Hz

Speed $v = \nu\lambda = 60 \times 3 = 180$ m/s. Hence, verifies the option (*c*).

(*d*) As waves are stationary so the amplitude of particles varies from 0 to $a = 0.06$ m from nodes to antinodes i.e., amplitude through out the wave are not constant. Rejects the option (*d*).

Q15.13. Speed of sound wave in a fluid depends upon

(*a*) directly on the density of the medium
(*b*) square of Bulk modulus of the medium
(*c*) inversely on the square root of density
(*d*) directly on the square root of Bulk modulus of the medium.

Ans. (*c, d*): Speed of sound wave in fluid of Bulk modulus k and density ρ is given by $v = \sqrt{\frac{k}{\rho}}$

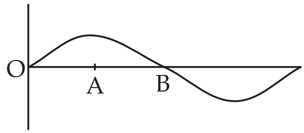
So $\quad v = \sqrt{k}$ (if ρ is constant)

and $\quad v = \sqrt{\frac{1}{\rho}}$ (if k is constant)

So verifies the options (*c*) and (*d*).

Q15.14. During the propagation of plane progressive mechanical wave
(*a*) all particles are vibrating in the same phase
(*b*) amplitude of all particles is equal
(*c*) particles of the medium executes SHM
(*d*) wave velocity depends upon the nature of medium.

Ans. (*b, c, d*): During propagation of mechanical wave each particles displaces from zero to maximum i.e., upto amplitude. So amplitude of each particle is equal. Verifies the option (*b*).



Each particle between any two successive crests and troughs are in different mode of vibration i.e., in different phase rejects the option (*a*).
For progressive wave medium particles oscillates about their mean position in which restoring force $F \propto (-y)$. So motion of medium particles is simple harmonic motion. So verifies the option (*c*).
For progressive wave propagating in a medium of density (ρ) and Bulk modulus k the velocity (v).

$$v = \sqrt{\frac{k}{\rho}}$$

As the v depends on k and ρ and k, ρ are different for different medium so v of wave depends on nature of medium, hence, verifies the option (*d*).

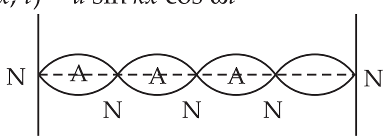
Q15.15. The transverse displacement of a string (clamped at it's both ends) is given by

$$y(x, t) = 0.06 \sin\left(\frac{2\pi}{3}x\right) \cos(120\pi t)$$

All the points on the string between two consecutive nodes vibrate with
(*a*) same frequency
(*b*) same phase
(*c*) same energy
(*d*) different amplitude

Ans. (*a, b, d*): Given $y(x, t) = 0.06 \sin\left(\frac{2\pi}{3}x\right) \cos(120\pi t)$ show the stationary waves, as the standard eqn. of stationary wave is

$$y(x, t) = a \sin kx \cos \omega t$$



(*a*) the frequencies of all particles are same, verifies the option (*a*).
(*b*) particles between any two consecutive nodes vibrates either upside or downside having same phase $120\pi t$ at a time, verifies the option (*b*).
(*c*) As the amplitude of different particles are different between two nodes and energy $(E) \propto A^2$. So particles have different energies. So rejects the option (*c*) and verifies the option (*d*).

Q15.16. A train is standing in station yard, blows the whistle of frequency 400 Hz in still air. The wind start blowing in the direction from yard to the station with speed of 10 m/s. Given that the speed of sound in still air is 340 m/s, then

(*a*) the frequency of sound as heard by an observer standing on the platform is 400 Hz.

(*b*) the speed of sound for the observer standing on the platform is 350 m/s.

(*c*) the frequency of the sound as heard by the observer standing on the platform will increase.

(*d*) the frequency of sound as heard by the observer standing on the platform will decrease.

Ans. (*a*, *b*): $\nu_0 = 400$ Hz frequency of source of sound.

Velocity of wind $v_w = 10$ m/s from source of listener.

Speed of sound in still air $= v_s = 340$ m/s.

As the listener is standing on platform.

Speed of sound with respect to listener $= v_s + v_w = 340 + 10 = 350$ m/s.

Verifies the option (*b*).

As the distance between listener and source does not change so frequency of sound does not change as heard by listener. i.e., he heard $\nu_0 = 400$ Hz. Verifies option (*a*), rejects the option (*c*) and (*d*) as it is constant $\nu_0 = 400$ Hz.

Q15.17. Which of the following statements are true for stationary waves?

(*a*) Every particle has a fixed amplitude which is different from the amplitude of it's nearest particle.

(*b*) All the particles cross their mean position at the same time.

(*c*) All the particles are oscillating with same amplitude.

(*d*) There is no net transfer of energy across any plane.

(*e*) There are some particles which are always at rest.

Ans. (*a, b, d, e*): In stationary waves [$y(x, t) = a \sin kx \cos \omega t$] the particles between two nodes vibrates with different amplitude which increases from node to antinodes from **zero to maximum**, and then decreases from **maximum to zero** from antinodes to nodes. The amplitude of a particle will remain constant $a \cos kx$, but varies with λ

$\because\ k = \dfrac{2\pi}{\lambda}$. Hence **verifies the option (*a*), rejects option (*c*)**, the amplitude of particles at nodes has amplitude zero verifies option (*e*).

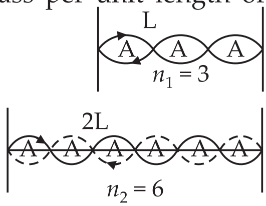
As the particles at nodes are rest so energy does not transfer verifies option (*d*).

Particles between two nodes are in same phase i.e., motion of particles between two nodes will be either upward or downward and crosses the mean position at same time. Hence verifies option (*b*).

VERY SHORT ANSWER TYPE QUESTIONS

Q15.18. A sonometer wire is vibrating in resonance with a tuning fork keeping the tension applied same, the length of the wire is doubled. Under what conditions would the tuning fork still be is resonance with the wire?

Ans. We know then when a wire of length L vibrates its resonant frequency in nth mode after stretching its by a tension T then frequency of nth harmonic is $\nu = \frac{n}{2L}\sqrt{\frac{T}{m}}$, here m is mass per unit length of stretched wire.



Let in given two cases

$$\nu_1 = \frac{n_1}{2L_1}\sqrt{\frac{T_1}{m_1}}$$

$$\nu_2 = \frac{n_2}{2L_2}\sqrt{\frac{T_1}{m_1}}$$

In given question $T_1 = T_2 = T$ (given)

$m_1 = m_2 = m$ as wire same

$L_2 = 2L_1$

$$\therefore \quad \frac{\nu_1}{\nu_2} = \frac{n_1\sqrt{T}\sqrt{m} \times 2 \times 2L_1}{n_2\sqrt{T}.\sqrt{m}\ 2L_1} = \frac{2n_1}{n_2}$$

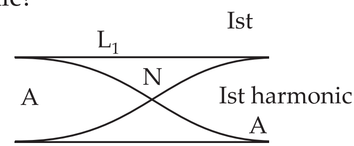
As tuning fork is same i.e., in both harmonics n_1 and n_2 frequency of resonance same $\therefore \nu_1 = \nu_2$ or $\frac{2n_1}{n_2} = 1$

$$n_2 = 2n_1$$

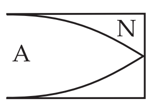
So when length of wire double the number of harmonics double for same resonant frequency, of tension and m.

Q15.19. An organ pipe of length L open at both ends is found to vibrate in its first harmonic when sounded with tuning fork 480 Hz. What should be the length of a pipe closed at one end so that it also vibrates in it's first harmonic with same harmonic?

Ans. As, the medium and frequency and number of harmonic in open and closed pipes are same, so number of nodes and λ (wave), in both cases will be same.



Both end open pipe



One end close pipe

In both end open pipe

$$L_1 = \frac{2 \times \lambda_1}{4} \quad \text{or} \quad \lambda_1 = 2L_1$$

and $\nu_1 = \frac{c}{\lambda_1}$, $\nu_1 = \frac{c}{2L_1}$

In one end open pipe

$$L_2 = \frac{1\lambda_2}{4} \quad \text{or} \quad \lambda_2 = \frac{c}{4L_2}$$

As medium and tuning fork in both cases are same $\nu_1 = \nu_2$ and $c_1 = c_2 = c$

$$\nu_2 = \frac{c}{4L_1} \quad \text{or} \quad \nu_1 = \nu_2$$

$$\frac{c}{2L_1} = \frac{c}{4L_2}$$

or $$4L_2 = 2L_1 \quad \text{or} \quad L_2 = \frac{L_1}{2}$$

So the length of one end closed pipe will be half of both end open pipe for resonant at Ist harmonic with same frequency.

Q15.20. A tuning fork 'A' marked 512 Hz produces 5 beats per second, when sounded with another unmarked tuning fork B. If B is loaded with wax. number of beats is again 5 per second. What is the frequency of tuning fork 'B' when unloaded.

Ans. $\nu_A = 512,\quad \because \nu_0 = \nu_A \sim \nu_B$ no. of frequency of beats

As on loading frequencies of B decreased

So in first case $\nu_1 = 5$

$$\nu_B = \nu_A \pm 5$$

When B is loads frequency of B is ν

$\therefore$ $\nu_B = 512 \pm 5$ i.e., ν_B either 507 or 517.

On load in frequency of B decreased 507 to lower value of number of beats will increase so $\nu_B \neq 507$. Now if $\nu_B = 517$ then its frequency decrease by 10 Hz then number of betas will also be same as $512 - 507 = 5$. So frequency of tuning fork when unloaded is **517**.

Q15.21. The displacement of elastic wave is given by the function $y = 3 \sin \omega t + 4 \cos \omega t$, where y is in cm and t is in second, calculate the resultant amplitude.

Ans. $\because$ $y = 3 \sin \omega t + 4 \cos \omega t$ ...(*i*)

Let $3 = a \cos \phi$ ...(*ii*) and $4 = a \sin \phi$ ...(*iii*)

Then $y = a \cos \phi \sin \omega t + a \sin \phi \cos \omega t$

$$y = a \sin (\omega t + \phi)$$

From (*ii*) and (*iii*)

$$\tan \phi = \frac{4}{3} \quad \text{or} \quad \phi = \tan^{-1} \frac{4}{3}$$

On squaring and adding (*ii*) and (*iii*) equations

$$a^2 \cos^2 \phi + a^2 \sin^2 \phi = 3^2 + 4^2$$

$$a^2(\cos^2 \phi + \sin^2 \phi) = 9 + 16$$

$$a^2 = 25 \quad \Rightarrow \quad a = 5$$

$$y' = 5 \sin (\omega t + \phi) \text{ when } \phi = \tan^{-1} \frac{4}{3}$$

Hence, New amplitude is 5 cm.

Q15.22. A sitar wire is replaced by another wire of same length and material, but of three times their earlier radius. If the tension remains same, by what factor will the frequency change.

Ans. The wire is stretched both end so frequency of stretched wire is

$$\nu = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

As number of harmonic n, length L and tension (T) are kept same in both cases. $\therefore \nu \propto \frac{1}{\sqrt{m}}$

$$\frac{\nu_1}{\nu_2} = \frac{\sqrt{m_2}}{\sqrt{m_1}} \quad \text{...}(i)$$

$$\text{Mass per unit length} = \frac{\text{mass of wire}}{\text{length}} = \frac{(\pi r^2 l)\rho}{l}$$

$$m = \pi r^2 \rho$$

As material of wire is same.

$$\frac{m_2}{m_1} = \frac{\pi r_2^2 \rho}{\pi r_1^2 \rho} = \frac{(3r)^2}{r^2} = \frac{9}{1}$$

$$\therefore \quad \frac{\nu_1}{\nu_2} = \sqrt{\frac{9}{1}} = \frac{3}{1} \quad \therefore \nu_2 = \frac{1}{3}\nu_1$$

So the frequency of sitar reduced by $\frac{1}{3}$ of previous value.

Q15.23. At what temperature (in °C) will the speed of sound in air be 3 times of its speed at 0°C?

Ans. $v \propto \sqrt{T}$ $\qquad \frac{v_T}{v_0} = \sqrt{\frac{T}{T_0}}$

$$v_T = 3v_0 \text{ (given)}$$

$$\therefore \quad \frac{3v_0}{v_0} = \sqrt{\frac{T}{273+0}} \quad \text{or} \quad \sqrt{T} = 3\sqrt{273}$$

$$T = 9 \times 273 = 2457 \text{ K}$$

$$\text{or} \quad T = 2457 - 273 = 2184°\text{C}.$$

Q15.24. When two waves of almost equal frequencies n_1 and n_2 reached at a point simultaneously. What will be the time interval between successive maxima?

Ans. (*b*): Here frequencies of vibrations are nearly equal but exactly different $n_1 \simeq n_2$ so beats are form in the medium when they produces sound, the number of let $n_2 > n_1$.

Then number of beats per second i.e., frequency of maximas = $n = n_2 - n_1$.

So time period of maxima or beats = $\frac{1}{n} = \frac{1}{n_2 - n_1}$ second.

SHORT ANSWER TYPE QUESTIONS

Q15.25. A steel wire has a length of 12 m and mass of 2.10 kg. What will be speed of transverse wave in this wire? When tension is 2.06×10^4 N is applied.

Ans. $l = 12$ m M (Total mass) = 2.10 kg

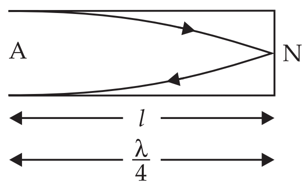
$$m = \frac{M}{l} = \frac{2.1}{12} \qquad T = 2.06 \times 10^4 \text{ N}$$

$$\therefore \quad v = \sqrt{\frac{T}{m}} = \sqrt{\frac{2.06 \times 10^4 \times 12}{2.10}} = \sqrt{\frac{1236 \times 10^4}{105}}$$

$$= \sqrt{11.77} \times 10^2 = 3.43 \times 10^2$$

$$v = 343.0 \text{ m/s.}$$

Q15.26. A pipe 20 cm is closed at one end. Which harmonic mode of the pipe is resonantly excited by source 1237.5 Hz (velocity of sound in air = 330 m/s)?



Ans. $l = 20$ cm = 0.2 m, $\nu = 1237.5$ Hz

$v = 330$ m/s

$l = \frac{\lambda}{4}$ or $\lambda = 4l$

for fundamental frequency

$$\therefore \quad \nu_1 = \frac{v}{\lambda} = \frac{v}{4l}$$

$$= \frac{330}{4 \times 20 \times 10^{-2}} = 412.5 \text{ Hz}$$

$$\nu \text{ (given)} = 1237.5 \text{ Hz}$$

$$\therefore \quad \frac{\nu(g)}{\nu_1} = \frac{1237.5}{412.5} = \frac{3}{1}$$

∵ The frequency in Ist, IInd, IIIrd ... harmonic are in the ratio 1 : 2 : 3 : 4 ... in one end open pipe. So there is IIIrd harmonic excited by 1237.5 Hz frequency.

Q15.27. A train standing at the outer signal of a railway station blows a whistle of 400 Hz in still air. The train begins to move with a speed of 10 m/s towards platform. What is the frequency of sound for an observer standing on the platform? (Sound velocity in air = 330 m/s)

Ans. $\nu_0 = 400$ Hz $v_s = 10$ m/s

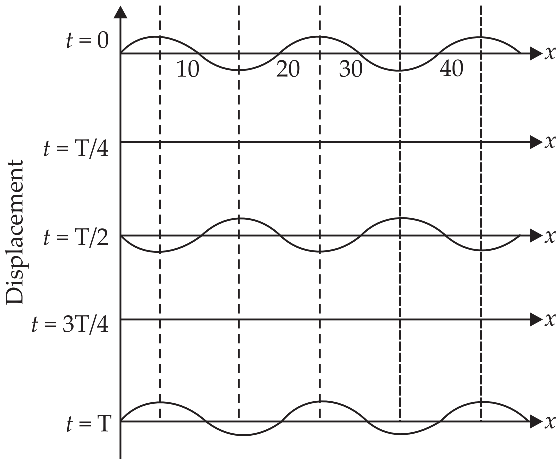
Velocity of sound in air $v_a = 330$ m/s

Apparent frequency by observer standing on platform

$$\nu' = \frac{v_a}{(v_a - v_s)} \nu_0 = \frac{330 \times 400}{(330 - 10)}$$

$$\nu' = \frac{330 \times 400}{320} = \frac{825}{2} = 412.5 \text{ Hz.}$$

Q15.28. The wave pattern on the stretched string is shown in figure. Interpret what kind of wave this is and find its wavelength.



Ans. The displacement of medium particles at distance 10, 20, 30, 40 and 50 cm are always rest which is the property of nodes in stationary wave.

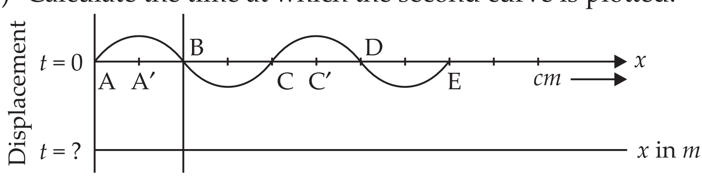
At $t = \frac{T}{4}$ and $\frac{3T}{4}$ all particles are at rest which is in stationary wave when the particle crosses its mean position.

So the graph of wave shows stationary wave. The wave at x = 10, 20, 30, 40 cm there are nodes and distance between successive nodes is $\frac{\lambda}{2}$

$$\therefore \quad \frac{\lambda}{2} = (30 - 20) \quad \text{or} \quad \lambda = 20 \text{ cm.}$$

Q15.29. The pattern of standing waves formed on a stretched string at two instant of time are shown in figure. The velocity of the two waves super-imposing to form stationary wave is 360 ms^{-1} and their frequencies are 256 Hz.

(*a*) Calculate the time at which the second curve is plotted.



(*b*) Mark nodes and anti-nodes on the curve.

(*c*) Calculate the distance between A′ and C′.

Ans. Given frequency of the wave ν = 256 Hz

$$\therefore \quad T = \frac{1}{\nu} = \frac{1}{256} \text{ second} = 0.00390$$

$$T = 3.9 \times 10^{-3} \text{ seconds.}$$

(*a*) In stationary wave a particle passes through it's mean position after every $\frac{T}{4}$ time

∴ In IInd curve displacement of all medium particle, are zero so

$$t = \frac{T}{4} = \frac{3.9 \times 10^{-3}}{4} = .975 \times 10^{-3} \text{ sec}$$

$$t = 9.8 \times 10^{-4} \text{ second.}$$

(*b*) Points does not vibrate i.e., their displacement is zero always so nodes are at A, B, C, D and E. The points A′ and C′ are at maximum displacements so there are anti-nodes at A′ and C′.

(*c*) At A′, C′ there are consecutive anti-nodes so the distance between A′ and C′ = $\lambda = \frac{v}{\nu} = \frac{360}{256} = \frac{90}{64} = 1.41$ m.

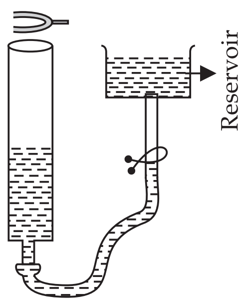
Q15.30. A tuning fork vibrating with a frequency of 512 Hz is kept close to open end of a tube filled with water (figure). The water level in the tube is gradually lowered.

When the water level is 17 cm below the open end, maximum intensity of sound is heard. If the room temperature is 20°C. Calculate:

(*a*) speed of sound in air at room temperature.

(*b*) speed of sound in air at 0°C.

(*c*) if the water in the tube is replaced with mercury, will there be any difference in your observations?



Ans. (*a*) Pipe filled partially with water or mercury behave like an one end open organ pipe. In first harmonic, one anti-node and one node at water level formed so, first harmonic or at maximum intensity is heard at L = 17 cm so

$$L = \frac{\lambda}{4} \quad \therefore \quad \lambda = 4L = 4 \times 17$$

$$\lambda = 68 \text{ cm} = 0.68 \text{ m}$$

$$v = \nu\lambda = 512 \times 0.68 \text{ m/s}$$

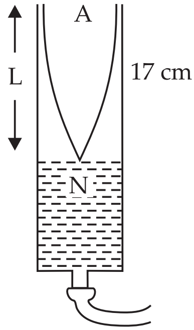
$$v = 348.16 \text{ m/s}$$

So velocity of sound in air at room temperature 20°C is 348.16 m/s (v_{20})

(*b*) $\because v \propto \sqrt{T} \quad \therefore \quad \frac{v_0}{v_T} = \sqrt{\frac{T_0}{T}}$

$T_0 = 273$ K

$v_T = v_{20} = 348.16 \qquad T = 20 + 273 = 293$ K



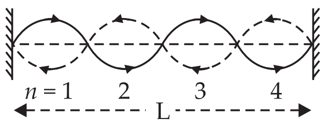
$$v_0 = v_T\sqrt{\frac{273}{293}} = 348.16\sqrt{0.9317}$$

$$v_0 = 348.16 \times 0.96526 = 336 \text{ m/s}$$

(*c*) Water and mercury in tube reflects the sound into air column to form stationary wave and reflection is more in mercury than water as mercury is more denser than water. So intensity of sound heard will be larger but reading does not change as medium in tube (air) and tuning fork are same.

Q15.31. Show that when a string fixed at its ends vibrates in 1 loop, 2 loops, 3 loops and 4 loops the frequencies are in the ratio 1 : 2 : 3 : 4.

Ans. let n be the number of loops in the string.



The length of each loop is $\frac{\lambda}{2}$

$$\therefore \quad L = \frac{n\lambda}{2} \quad \text{or} \quad \lambda = \frac{2L}{n}$$

$$v = \nu\lambda \quad \text{and} \quad \lambda = \frac{v}{\nu}.$$

So $$\frac{v}{\nu} = \frac{2L}{n}$$

$$\nu = \frac{n}{2L}.v$$

v in stretch string $= \sqrt{\frac{T}{m}}$

$$\therefore \quad \nu = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

For $n = 1$, $$\nu_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} = \nu_0$$

If $n = 2$ then $$\nu_2 = \frac{2}{2L}\sqrt{\frac{T}{m}} = 2\nu_0$$

$n = 3$ then $$\nu_3 = \frac{3}{2L}\sqrt{\frac{T}{m}} = 3\nu_0$$

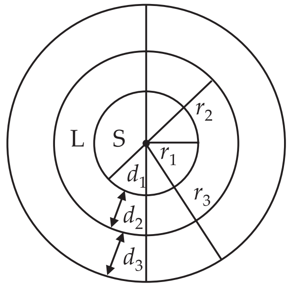
$\therefore \nu_1 : \nu_2 : \nu_3 : \nu_4 = n_1 : n_2 : n_3 : n_4 = 1 : 2 : 3 : 4$

LONG ANSWER TYPE QUESTIONS

Q15.32. The earth has the radius 6400 km. The inner core of 1000 km radius is solid. Outside it there is a region from 1000 km to a radius 3500 km which is in molten state. Then again 3500 km to 6400 km the earth is solid. Only longitudinal (P) waves can travel inside a liquid.

Assume that P waves have a speed of 8 km/second in solid part and of 5 km/second in liquid part of earth. An earthquake occurs at some place close to the surface of earth. Calculate the time after which it will be recorded in a seismometer at a diametrically opposite point on the earth if wave travels along diameter.

Ans. $r_1 = 1000$ km
$r_2 = 3500$ km
$r_3 = 6400$ km
$d_1 = 1000$ km
$d_2 = 3500 - 1000 = 2500$ km
$d_3 = 6400 - 3500 = 2900$ km



Solid distance diametrically

$$= 2(d_1 + d_3) = 2(1000 + 2900)$$
$$= 2 \times 3900 \text{ km}$$

Time taken by wave produced by earthquake in solid part

$$= \frac{3900 \times 2}{8} \text{ sec}$$

Liquid part along diametrically $= 2d_2 = 2 \times 2500$

$\therefore$ Time taken by seismic wave in liquid part $= \dfrac{2 \times 2500}{5}$

$$\text{Total time } = \frac{2 \times 3900}{8} + \frac{2 \times 2500}{5} = 2\left[\frac{3900}{8} + \frac{2500}{5}\right]$$
$$= 2[487.5 + 500] = 2 \times 987.5 = 1975 \text{ sec.}$$
$$= 32 \text{ min } 55 \text{ sec.}$$

Q15.33. If c is the r.m.s. speed of molecules in a gas and v is the speed of sound wave in the gas. show that $\frac{c}{v}$ is constant and independent of temperature for all diatomic gases.

Ans. We know that $c = \sqrt{\dfrac{3P}{\rho}}$ for molecules.

$$c = \sqrt{\frac{3RT}{M}}$$

$$\therefore \quad \frac{P}{\rho} = \frac{RT}{M} \qquad \because \frac{P}{\rho} = \frac{RT/V}{M/V}$$

$\because$ PV $= n$RT
$n = 1$
$$P = \frac{RT}{V}$$

M = molar mass of gas

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

$$\frac{c}{v} = \frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{\gamma RT}{M}}} = \sqrt{\frac{3}{\gamma}}$$

$$\gamma = \frac{C_p}{C_v} = \text{adiabatic constant for diatomic gas}$$

$$\gamma = \frac{7}{5}$$

$$\therefore \quad \frac{c}{v} = \sqrt{\frac{3}{7/5}} = \sqrt{\frac{15}{7}} = \text{constant.}$$

Q15.34. Given below are some functions of x and t to represent the displacement of an elastic wave.

(*i*) $y = 5\cos(4x)\sin 20t$

(*ii*) $y = 4\sin\left(5x - \frac{t}{2}\right) + 3\cos\left(5x - \frac{t}{2}\right)$

(*iii*) $y = 10\cos[(252 - 250)\pi t]\cos[(252 + 250)\pi t]$

(*iv*) $y = 100\cos[100\pi t + 0.5x)$

State which of these represent

(*a*) a travelling wave along $(-x)$ direction

(*b*) a stationary wave (*c*) beats

(*c*) a travelling wave along $(+x)$ direction.

Give reasons for the answers.

Ans. (*a*) A travelling wave along $(-x)$ direction must have $+kx$ i.e., in (*iv*) $y = 100\cos(100\pi t + 0.5x)$ so (*a*) (*iv*).

(*b*) A stationary wave of the for $y = 5\cos(4x)\sin 20t$ is a stationary wave so (*b*) (*i*).

(*c*) Beats involve $(\nu_1 + \nu_2)$ and $(\nu_1 - \nu_2)$ so beats can be represented by $y = 10\cos[(252 - 250)\pi t]$ represents beat so (*c*) (*iii*).

(*d*) $$y = 4\sin\left(5x - \frac{t}{2}\right) + 3\cos\left(5x - \frac{t}{2}\right) \quad ...(i)$$

Let $4 = a\cos\phi$ ...(*ii*) and $3 = a\sin\phi$ ...(*iii*)

$a^2\cos^2\phi + a^2\sin^2\phi = 4^2 + 3^2$ squaring and adding (*ii*), (*iii*)

$a^2 = 25 \Rightarrow a = 5$

Substituting (*ii*), (*iii*) in (*i*)

$$y = a\cos\phi\sin\left(5x - \frac{t}{2}\right) + a\sin\phi\cos\left(5x - \frac{t}{2}\right)$$

$$y = a\sin\left(5x - \frac{t}{2} + \phi\right)$$

$$y = 5\sin\left(5x - \frac{t}{2} + \phi\right)$$

Which represents the progressive wave in + x direction as the sign of kx (or $5x$) and $\omega t\left(\frac{1}{2}t\right)$ are opposite so it travels in + x direction. So (*d*) (*ii*)

Q15.35. In given progressive wave $y = 5\sin(100\pi t - 0.4\pi x)$, where x, y is in m and t in seconds, what is the

(*a*) amplitude
(*b*) wavelength
(*c*) frequency
(*d*) wave velocity
(*e*) particle velocity amplitude

Ans. Standard form of progressive wave travelling in + x direction (kx and ωt have opposite sign is given)

eqn. is $$y = a\sin(\omega t - kx + \phi)$$
$$y = 5\sin(100\pi t - 0.4\pi x + 0)$$

(*a*) Amplitude $a = 5$ m

(*b*) Wavelength λ, $k = \frac{2\pi}{\lambda}$

$$k = 0.4\pi$$

$$\lambda = \frac{2\pi}{k} = \frac{2\times\pi}{0.4\pi} = 5 \text{ m}$$

(*c*) Frequency ν, $\omega = 2\pi\nu \Rightarrow \nu = \frac{\omega}{2\pi}$ $\quad\because \omega = 100\pi$

$$\therefore \quad \nu = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

(*d*) Wave velocity $v = \nu\lambda = 50 \times 5 = 250$ m/s

(*e*) Particle (medium) velocity in the direction of amplitude at a distance x from source.

$$y = 5\sin(100\pi t - 0.4\pi x)$$

$$\frac{dy}{dt} = 5 \times 100\pi\cos(100\pi t - 0.4\pi x)$$

For maximum velocity of particle is at its mean position

$\cos(100\pi t - 0.4\pi x) = 1$

$\Rightarrow 100\pi t - 0.4\pi x = 0$

$$\therefore \left(\frac{dy}{dt}\right)_{max} = 5 \times 100\pi \times 1$$

v_{max} of medium particle = 500π m/s.

Q15.36. For the harmonic travelling wave

$$y = 2\cos 2\pi(10t - 0.0080x + 3.5)$$

where x and y are in cm and t is in second. What is the phase difference between the oscillatory motion at two points separated by a distance of (*a*) 4 m (*b*) 0.5 m

(*c*) $\frac{\lambda}{2}$ (*d*) $\frac{3\lambda}{4}$ at given instant of time

(*e*) What is the phase difference between the oscillation of the particle located at $x = 100$ cm at $t = T$ second and $t = 5$ s?

Ans.

$$y = 2 \cos 2\pi (10t - 0.0080x + 3.5)$$

$$y = 2 \cos (20\pi t - 0.016\pi x + 7.0\pi)$$

Wave is propagated in + x direction because ωt and kx are in with opposite sign standard equation $y = a \cos (\omega t - kx + \phi)$

$a = 2$, $\omega = 20\pi$, $k = 0.016\pi$ and $\phi = 7\pi$

(*a*) Path difference $p = 4$ m (given) = 400 cm

phase difference $\Delta\phi = \frac{2\pi}{\lambda} \times p = \frac{2\pi}{\lambda} \times 400$

$$\Delta\phi = k \times 400 = 0.016\pi \times 400$$

phase difference $\Delta\phi = 6.4\pi$ rad.

(*b*) Path difference $p = 0.5$ m = 50 cm

$$\Delta\phi = kp = 0.016\pi \times 50 = 0.8\pi \text{ rad.}$$

(*c*) Path difference $p = \frac{\lambda}{2}$

$$\Delta\phi = \frac{2\pi}{\lambda} p = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi \text{ radian}$$

(*d*)

$$\Delta\phi = \frac{2\pi}{\lambda} p = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3}{2}\pi \text{ radian}$$

(*e*)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{20\pi} = \frac{1}{10} \text{ sec}$$

$$x = 100 \text{ cm}$$

$$t = T$$

At $x = 100$, $t = T$

$\phi_1 = 20\pi T - 0.016\pi (100) + 7\pi = 20\pi \times \frac{1}{10} - 1.6\pi + 7\pi = 7.4\pi$

At $t = 5$ s

$\phi_2 = 20\pi(5) - 0.016\pi (100) + 7\pi = 100\pi - 1.6\pi + 7\pi = 105.4\pi$

$\phi_2 - \phi_1 = 105.4\pi - 7.4\pi = 98\pi$ radian.

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